Report on the outcomes of a Short-Term Scientific Mission[[1]](#footnote-1)

Action number: CA18232

Grantee name: Catherine Drysdale

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| **Details of the STSM**  Title: Title: Quasi-basis of a Metric Graph  Start and end date: 03/03/2024 to 04/03/2024 |
| **Description of the work carried out during the STSM**  Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section. |
| *(max. 500 words)*  The objectives of the STSM were to perform a literature review, decide on a test case for computations and following this do the computations. However, as metric graphs were a new topic for the applicant, so the host spent time introducing the topic to the applicant and getting her up to speed. To this end, knowledge transfer filled the majority of the STSM. However, in light of this, the first two objectives were completed during the STSM. The computations will be done at a later point and will be outlined in the planned activities.  In the original application, we considered a Laplacian with an additional drift term on , which gave rise a quasi-basis structure that allowed us to say the spectrum was similar to a self-adjoint operator providing we placed addition assumptions on the domain on the domain of the original problem. In the context of the literature, we decided to focus on non-self-adjointness via the boundary conditions on the Laplacian instead of via a drift term. In this way, we are aligned with the literature on non-self-adjoint metric graphs that can be found in the references; “Non-self-adjoint Graphs” Hussein et al. 2013, and “Hidden Symmetries in Non-self-adjoint Graphs” Hussein et al. 2013.  During on the STSM, we reconsidered the following test case that can be found in “Non-self-adjoint Graphs”, let be a graph consisting of two external edges and one vertex . Identifying the graph with the real line and the vertex with zero, we consider the following boundary conditions  and .  This operator is interesting because it is similar to the self-adjoint Laplacian via the similarity transformation  ,  where the exact form can be found in the article.  To see where the synergy is between these kinds of transforms and the transforms found in “*Partial inner product spaces: theory and applications,”* Antoine and Trapani 2009, we have to consider what these transformations would be in the context of a lattice of Hilbert Spaces. The lattice of Hilbert Spaces that is motivated by the drift term has consistent boundary conditions across the first half of the lattice, but the resulting lattice may be intriguing and particular than this. |
| **Description of the STSM main achievements and planned follow-up activities**  Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.  *(max. 500 words)*  The achievements of the STSM were outlined in the last section in the description of the work carried out. In the applicant’s opinion, the most important achievement was knowledge transfer between the applicant, which has laid the foundation for an exciting future project that is interconnected with the project in the applicant’s previous STSMs.  However, as outlined in the above section, what is missing are the numerical goals that were outlined in the original application, namely “we will then perform verified spectral pseudospectral computations. Eigenvalue problems on text works often give rise to nonlinear eigenvalue problems. Therefore, it is possible to by using the inf-beyn computations also.” We have not necessarily done the theoretical work to get to the point of doing spectral computations yet, but we have put a working plan in place that includes regular meetings between the host and the applicant. The jobs that will be done for review in these online meeting will be the completion of literature review and the generation of the lattice, following this we will do appropriate computations. The follow up activities will be directed towards a publication in an appropriate journal and conferences.  We have also facilitated upcoming work by the host’s securing of additional funds for the applicant to revisit Hagen. Additionally, we have secured a virtual mobility grant via the cost action as part of a larger project, of which the applicant and host are a part, looking at perturbations of brain networks. We believe that we can apply some of these ideas discussed in this STSM, such as representing changes to boundary conditions via perturbations to be applied to this project. |

1. This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant. [↑](#footnote-ref-1)