

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA18232

Grantee name: Sahiba Arora

Details of the STSM

Title: Study of the Ornstein-Uhlenbeck semigroups on compact infinite graphs - Part II

Start and end date: 01/05/2022 to 17/05/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

The purpose of this STSM was to start a collaboration with the host Assist. Prof. Dr. Marjeta Kramar Fijavž. During my stay, I also had the opportunity to interact with other members of the Department of Mathematics at the University of Ljubljana, in particular, Assist. Prof. Dr. Marko Kandić. Moreover, I was invited to give a talk at the Algebra and functional analysis seminar of the Faculty of Mathematics and Physics which gave me the opportunity to present my research to the members of the department.

At the beginning of my stay, we discussed the current known results about the semigroup generated by the Ornstein-Uhlenbeck operator and decided on the three main aspects that we would like to study. We also realized that the semigroup has not been studied on compact star graphs (or even compact intervals) before. Despite not being interesting in its own right – the knowledge of the properties on compact intervals seemed crucial for our investigation on rooted trees. Our idea was to gain an understanding on compact graphs and combine this with the techniques of Mugnolo and Rhandi – who studied the semigroup on star graphs – to obtain worthwhile results to rooted trees. In addition, we discussed the possibility of studying the semigroup on L^∞ where we expected the semigroup to be bi-continuous.

¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

During the first week of my stay, we had a virtual meeting with Prof. Dr. Abdelaziz Rhandi to discuss the three aspects mentioned above. We discussed the suitability of the idea that the decomposition of the state space into the symmetric and anti-symmetric parts reduces the Ornstein-Uhlenbeck operator, as it was done for the Laplacian in "K. Naimark and M. Solomyak: Geometry of the Sobolev spaces on the regular trees and Hardy's inequalities, Russ. J. Math. Phys. 8, 322–35 (2001)". For bicontinuous semigroups, we discussed the possibility to show generation via perturbation techniques or the Lumer-Phillips theorem for bi-continuous semigroups.

During the second week, we were able to show generation for bi-continuous semigroups on star graphs using the Lumer-Phillips theorem mentioned above. The approach also made it clear that it can be adapted to rooted trees with the knowledge of the unique invariant measure. Further, we had a virtual meeting with Prof. Dr. Delio Mugnolo and discussed the choice of the invariant measure. The techniques discussed during this meeting eventually turned out to be suitable and we were able to find the measure for the case of regular radially symmetric rooted trees.

After the completion of the STSM, I have had two meetings with Assist. Prof. Dr. Marjeta Kramar Fijavž, Prof. Dr. Delio Mugnolo, and Prof. Dr. Abdelaziz Rhandi. The next step in our analysis is to study the spectral properties and also impose Wentzell boundary conditions instead of Kirchoff and obtain similar results. We expect that an article concerning the Ornstein-Uhlenbeck semigroup will be ready for submission in a few months.