

## Report on the outcomes of a Short-Term Scientific Mission<sup>1</sup>

Action number: 18232

Grantee name: Vyacheslav Pivovarchik

### **Details of the STSM**

Title: Spectra of noncompact quantum graphs with leads

Start and end date: 14/08/2022 to 28/08/2022

### **Description of the work carried out during the STSM**

Prof. Mugnolo and me (Prof. Pivovarchik) have been working intensively on an article "Distinguishing co-spectral graphs by scattering", which will be submitted to a journal soon. The starting point is the insight from 2001, which goes back to von Below, that there are isospectral metric graphs - metric graphs that cannot be distinguished with the help of their spectrum with respect to the canonical Laplace operator. Even more general classes of isospectral graphs were discovered by Butler and Grout in 2011. Since then, the question has been whether such metric graphs can be distinguished using other quantities after all. The approach that Prof. Mugnolo and I have now followed is based on scattering theory: What happens if you send an electromagnetic wave against the graph and investigate its scattering? So we were able to present pairs of isospectral graphs that behave differently under scattering at distinguished corners.

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### **Description of the STSM main achievements and planned follow-up activities**

- 1) We proved that in case of the scattering problem generated by the Sturm-Liouville equation with certain conditions on its potentials on a graph which consist of a lead (a ray) attached to a compact subgraph with generalized Neumann conditions at the vertices possesses essential (continuous) spectrum which covers the nonnegative half-axis and may have normal (isolated Fredholm) eigenvalues as well as eigenvalues embedded into the essential spectrum.
- 2) We showed that the number of normal eigenvalues in such problem is finite while the number of embedded eigenvalues may be infinite.
- 3) We found conditions under which the scattering S-function is meromorphic.

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<sup>1</sup> This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

- 4) We investigated the asymptotics of the S-function and showed that the main term of these asymptotics does not depend on the potentials of the Sturm-Liouville equation but only on the shape of the graph.
- 5) We showed how to find the shape of the graph using the asymptotics of the scattering S-function and the location of eigenvalues at least in case where the number of the vertices does not exceed 6.

The STSM achieved its planned goals. The possibility to distinguish iso-spectral metric graphs using the scattering data obtained by scattering experiment in the same graph equipped with a lead (an infinite edge) was shown for examples of graphs. The obtained results will be published soon. In future we plan to investigate whether non-isometric graphs exist having the same scattering data.

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