Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Mohamed Fkirine

Details of the STSM

Title: An LQ problem for the heat equation on networks with vertex-noise and vertex-control

Start and end date: 30/04/2023 to 29/06/2023

Description of the work carried out during the STSM

The study titled "*On the parabolic Cauchy problem for quantum graphs with vertex noise*" by Mihály Kovács and Eszter Sikolya, published in *Electron. J. Probab. (volume 28, pages 1-20, 2023)*, examines the Heat equation associated with quantum graphs and perturbed vertex conditions. The vertex conditions involve standard continuity and Kirchhoff assumptions at each vertex. By introducing additive Gaussian noise to the Kirchhoff conditions, the authors successfully establish the existence and uniqueness of a mild solution with continuous paths in the standard state space of square integrable functions on the edges.

The main objective of the STSM (Short Term Scientific Mission) was to investigate the asymptotic behavior of the solutions. Specifically, the aim was to study the strong Feller property of the transition semigroup associated with the solution. This investigation directly pertains to the boundary null controllability of the heat equation on quantum graphs with Kirchhoff boundary control. To tackle this problem, we transform the control problem into an equivalent problem of moments. This approach has previously been utilized by R. D. Russell and H. O. Fattorini in their work titled "*Exact controllability theorems for linear parabolic equations in one space dimension*," published in A*rch. Rat. Mech. Anal., 43 (1971), 272-292*.

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[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description of the STSM main achievements and planned follow-up activities

To investigate the null controllability of the Heat equation associated with quantum graphs mentioned earlier, our initial focus was on the specific case of a star graph. However, it was surprising to discover that the problem is not null controllable, despite the observed null controllability in the heat equation with Dirichlet boundary control. Nevertheless, we propose that by modifying the Kirchhoff boundary conditions to Neumann boundary conditions, it may be possible to achieve null controllability for the equation. Moving forward, our next objective is to explore this problem within the context of general graphs.

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