Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Lorenzo Lamberti

Details of the STSM

Title: Frustrated Ferromagnetic Spin Chains on the 3d-Sphere: A Variational Approach to Chirality Transitions

Start and end date: 09/01/2022 to 31/01/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

The main purpose of this STSM was to take a visit at the Department of Mathematics of the Technische Universität München (Munich) in Germany in order to keep working with the host, Prof. Dr. Marco Cicalese, and Dr. Andrea Kubin. This collaboration started on October 1st, 2021.

During my stay, I had fruitful discussions with the reaserch group. We studied the energy E_n of a ferromagnetic/antiferromagnetic frustrated spin system with values on two separated circumferences of the 3-dimensional unit sphere. It consists in the sum of a term E_n^0 that depends on the nearest and next-to-nearest interactions and a term P_n that counts the jumps of the spin from one circumference to the other one. Its form is the following:

$$E_n(u) = E_n^0(u) + P_n(u) = \sum_{i \in \mathbb{Z}_n(I)} \lambda_n\left[-\alpha\left(u^i, u^{i+1}\right) + \left(u^i, u^{i+2}\right)\right] + \sum_{i \in \mathbb{Z}_n(I)} \lambda_n k_n S(u),$$

where $\mathbb{Z}_n(I)$ is a lattice on the thorus $I = (0,1)$ made of equidistant points with distance λ_n approaching to 0 as the number n of the particles diverges, u is the spin of the system with values u^i parametrized by $i \in \mathbb{Z}_n(I)$, α is the frustration parameter and k_n is a divergent sequence.

The main purpose was to show how much energy the system spends for any magnetic anisotropy transitions and chirality transition by studying the asymptotic behaviour of the energy, that is when the system is close to the helimagnet/ferromagnet transition point $\alpha = \alpha_n \sim 4$ from below as the number of particles diverges.

During our discussions we also found the correct scalings and rinormalizations of the energy to detect the two fenomena.

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

As we expected, in order to calculate the energy that the system spends for magnetic anisotropy transitions (jumps between the two circumferences) and for chirality transitions, we computed the Γ-limit of its renormalizations at first and second order. At first order, which means considering the energy

$$H_n(u) = E_n(u) - min\ E_n,$$

we restrict every spin u on some intervals I_j that partition I such that on $\mathbb{Z}_n(I_j)$ it takes values in only one circumference. We eventually modify such restrictions in a way that they are well-connected on the boundary of the thorus I_j, denotating them as u_{I_j}. The functional H_n can be split in two terms:

$$H_n(u) = \sum_j \left[MM_n\left(u_{I_j}\right) + R_{n,j}(u)\right]$$

As long as we consider a remainder $R_{n,j}$ for each modification of the spin, the analysis of the global process can be localized in each circumference with the associated energy $MM_n\left(u_{I_j}\right)$.

At scale λ_n no chirality transitions arise and if the number of magnetic anisotropy transitions is finite, the rescaled energy H_n detects them and spends a constant value for each jump. At the second order we deal with the energy

$$H_n(u) - \sum_j R_{n,j}(u) = \sum_j MM_n\left(u_{I_j}\right).$$

If the number of magnetic anisotropy transitions is finite, we transpose the 3-dimensional problem into a finite number of 2-dimensional ones, whose analysis was lead in

- Cicalese, Marco; Solombrino, Francesco. Frustrated ferromagnetic spin chains: a variational approach to chirality transitions. Journal of Nonlinear Science, 25.2: 291-313, (2015).

The scale we need to compute the energy for chirality transitions is $\lambda_n\left(1-\frac{\alpha_n}{4}\right)^{\frac{3}{2}}$.

In the future we will extend the same problem in a 2-dimensional setting. We expect the research activity explained above to lead to the publication of a paper on an international mathematical journal.

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