

## Report on the outcomes of a Short-Term Scientific Mission<sup>1</sup>

Action number: CA18232

Grantee name: Marianna Porfido

### **Details of the STSM**

Title: Elliptic operators with unbounded coefficients: kernel estimates and maximal regularity

Start and end date: 09/01/2022 to 31/01/2022

### **Description of the work carried out during the STSM**

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

The purpose of this STSM was to continue the collaboration with the host, Dr. Markus Kunze, started on October 1st, 2021. During my stay I also had the chance to get in touch with the research group at the Department of Mathematics and Statistics of the University of Konstanz.

We were working with differential operators of the form  $Au = \text{div}(QDU) + FDU - Vu$  where the coefficients  $Q$ ,  $F$  and  $V$  are Hölder continuous,  $V$  is nonnegative and the matrix  $Q$  is symmetric and uniformly elliptic. Note that all coefficients may be unbounded near infinity. It is not difficult to see that a realization of  $A$  generates a semigroup  $T = (T(t))$  that is given through kernels  $p(t, x, y)$ . Thus, the underlying parabolic equation can be solved in a generalized sense.

Our main objective was to establish estimates for the derivatives of the transition kernel in the case of unbounded diffusion coefficients and deduce the regularity of the solutions of the associated problems. Our strategy was to look at the case of bounded diffusion coefficients, considered for example in the article "G. Metafun, D. Pallara, A. Rhandi, *Global properties of transition probabilities of singular diffusions, Theory Probab. Appl. 54 (2010), 68-96*", where the main tools are time dependent Lyapunov functions. Once estimates for the derivatives of the transition kernels have been found, we planned to extend these to unbounded diffusion coefficients by means of approximation.

### **Description of the STSM main achievements and planned follow-up activities**

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

As expected, estimates currently available in the literature for  $\|Dp\|_\infty$  in the case of bounded diffusion coefficients cannot be used in the approximation procedure, as the constants depend on  $\|q_{ij}\|_\infty$ . For this

<sup>1</sup> This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

reason, we made use of techniques such as the De Giorgi regularity method, as in the article “M. Kunze, L. Lorenzi and A. Rhandi: Kernel estimates for nonautonomous Kolmogorov equations. *Advances in Mathematics* 287 (2016), 600-639” to obtain estimates independent of  $\|q_{ij}\|_\infty$ . Another key point was to prove that  $\frac{Dp}{p^{1/2}}$  belongs to the space  $L^2$  and its  $L^2$ -norm can be controlled with terms that do not depend on the diffusion coefficients.

With these estimates established, we could now approximate (possibly unbounded) diffusion coefficients with bounded ones and prove convergence of all the terms appearing in the estimate, thus establishing the desired estimates in the general case. A slight modification yields estimates for  $\|wDp\|_\infty$  where  $w$  is a weight function. This gives more refined results on the transition kernels.

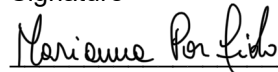
We currently apply our abstract results to obtain estimates for specific model operators involving polynomially or exponentially growing diffusion coefficients. We expect that an article summarizing the obtained results (which we are currently working on) will be ready for submission within a few months.

**The STSM grantee**

Marianna Porfido, PhD student  
Department of Mathematics  
University of Salerno  
Italy

Date: 16/02/2022

Signature

  
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**The host**

Dr. Markus Kunze  
Department of Mathematics and Statistics  
University of Konstanz  
Germany

Date: February 2<sup>nd</sup> 2022

Signature

  
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