

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA18232

Grantee name: Patrizio Bifulco

Details of the STSM

Title: Heat Kernels and Spectral Geometry on Quantum Graphs

Start and end date: 16/10/2023 to 31/10/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

During my visit at Stockholm University, I worked together with Prof. Dr. Pavel Kurasov mostly on spectral properties of quantum graphs (that is, considering differential operators on certain networks) paying particular attention to the gaps between the spectrum of the free Laplacian compared to the spectrum of the magnetic Laplacian having a so-called magnetic potential. This was indeed inspired by a previous work by myself, where I studied those gaps in the case for the free Laplacian and the usual Schrödinger operator on a metric graph, respectively. In fact, Prof. Kurasov and I were able to transfer some of those ideas to the setup of magnetic Laplacians, which mainly used quite interesting properties of the heat kernel such as small time asymptotics or Gaussian estimates. Our work basically started with the study of basic properties of magnetic Laplacians. We began our work primarily by studying basic references and, above all, deriving appropriate properties of the heat kernel in this situation. This enabled us, using the initial ideas in the setup of Schrödinger operators, to create a first formula for the average of the eigenvalue distances between the spectrum of the free Laplace and that of the magnetic Laplace. In fact, this formula also provides an Amartsumian-like theorem on networks, which was previously only known for electric potentials. Furthermore, we also worked on the existing heat content formula in order to optimize it, especially in its expression. Also here, we were able to study some new properties of the associated heat kernel.

Not only was I able to work with Prof. Kurasov. Together with Jonathan Rohleder, an associate professor at Stockholm University, I was able to work on the topic of the so-called Krein-Laplace on networks. An important aspect of our work was to study the corresponding heat kernel for this operator in more detail and, if possible, compare it with known results for heat kernels of usual Schrödinger operators (including the explorations of Prof. Kurasov and me during my visit). We managed to derive a short-term asymptotic for this heat kernel and also show that even in the case of Krein-Laplace there is a heat kernel at all. We then continued our work by deriving spectral properties of the Krein-Laplace, in particular in order to be able to derive lower and upper bounds for the eigenvalues of the Krein-Laplace. Above all, we began to better understand the principles of Krein-Laplace by reading an

¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

already existing article by Dr. Rohleder (together with Jacob Muller), in which some results of Krein-Laplace were already listed on networks.

Description of the STSM main achievements and planned follow-up activities

Thus, through the STSM I was able to achieve many of the goals I had previously set: With Prof. Kurasov, I was able to understand the properties of magnetic Laplacians on networks much better and, above all, learn properties from it that were previously unknown to me and which will also be very helpful in my further upcoming research. In addition, by working with Prof. Kurasov, I was able to deepen my knowledge of the heat content and significantly optimize some of my existing results in particular the explicit formula for the heat content. Prof. Kurasov and I are now in further contact in order to prove the assumed mean value formula for the spectral gaps in the case of magnetic potentials in full generality.

Also the collaboration with Dr. Rohleder was very fruitful in these matters. We were able to derive a variety of additional, previously unknown (particularly spectral) properties of the Krein-Laplace, so that we were able to set the first 15 pages of a joint research article. My doctoral supervisor, Prof. Delio Mugnolo, will also be involved in this work in the future and we are currently in joint discussion about our article. Furthermore, Dr. Rohleder and I managed to derive a first upper bound for the eigenvalues of the Krein-Laplace and also generalized some results to a somewhat more general class of operators containing the Krein-Laplace. Our collaboration also led to a (maximal!) class of operators (including also usual Schrödinger operators) for which a corresponding small-time asymptotic of the underlying heat kernel holds.

(max. 500 words)

In summary, I am very satisfied with my research stay at Stockholm University and, apart from Prof. Kurasov and Dr. Rohleder I got to know many other interesting personalities, also coming from many other mathematical disciplines. I also had the honor of presenting a research article written in collaboration with a co-author from Hagen in the official analysis seminar of the mathematical institute at Stockholm university, which also led to many other interesting mathematical conversations, which in particular also gave rise to further new questions for my upcoming research. For me it is also particularly important to maintain the contact with Prof. Kurasov and Dr. Rohleder, as my visit to Stockholm only laid the foundation for our further collaboration. I am very grateful that I was able to take advantage of this opportunity, supported by COST, and I would take this opportunity again at any time. I learned a lot of new things during this time!