Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232 [WG1]

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Details of the STSM

Title: **Staffans-Weiss perturbations for linear stochastic Cauchy problems**

Start and end date: **29/09/2022 to 28/10/2022**

Description of the work carried out during the STSM

In this project, we study the well-posedness and the large time behaviour of linear stochastic evolution equations under unbounded perturbations of the generator. More precisely, we considered the following linear perturbed stochastic evolution equations

$$(\textbf{SCP})_{\textbf{A+BKC,B}} \qquad \textbf{dX(t)} = (\textbf{A} + \textbf{BKC})\textbf{X(t)dt} + \textbf{BdW(t)}$$

in a Hilbert space H, where A is the generator of the strongly continuous semigroup $\left(T(t)\right)_{t\geq 0}$, $C \in L(D(A), Y)$ is a linear bounded operator (not necessarily closed or closable) from $D(A)$ to the Hilbert space Y, $B \in L(U, V)$ is a bounded operator from the separable Hilbert space U to V, where V is the extrapolation space associated to A and H, $K \in L(Y, U)$ is a bounded operator and $\left(W(t)\right)_{t\geq 0}$ is a cylindrical Wiener process over U.

Description of the STSM main achievements and planned follow-up activities

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During the STSM, the collaborators showed that under conditions on the triplet (A,B,C) the perturbed equation $(SCP)_{A+BKC,B}$ has a unique mild solution whenever the unperturbed problem

$$(SCP)_{A,B} \qquad dX = AX(t)dt + BdW(t)$$

Has a mild solution. Moreover, a variation of constant formula of the solutions of $(SCP)_{A+BKC,B}$ in terms of the semigroup $(T(t))_{t \geq 0}$ is established. On the other hand, they proved that the distributions of solutions of $(SCP)_{A,B}$ and $(SCP)_{A+BKC,B}$ are equivalents. Finally, the collaborators gave a sufficient condition for the existence of the invariant measure of the perturbed problem $(SCP)_{A+BKC,B}$.