

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA18232 - Mathematical models for interacting dynamics on networks

Grantee name: SALVARANI Francesco

Details of the STSM

Title: Study of coagulation-fragmentation-diffusion equations on networks

Start and end date: 24/10/2023 to 31/10/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

The mobility at the Spanish Institute of Mathematical Sciences (ICMAT) has been devoted to the study of fragmentation-diffusion equations on networks.

In particular, with Dr. Marco Antonio Fontelos López, we have studied the reciprocal influence of the fragmentation operator and of a diffusion operator depending not only on the space variable, but also on the size of the particles. Coagulation being described by a nonlinear operator, we have chosen to handle first the pure fragmentation case.

Some physical considerations impose a very demanding constraint on the diffusion operator, which is, in general, very singular in the size variable.

We have firstly studied the existence of self-similar profiles on a line and of explicit solutions in an elementary arc of a network. In some particular cases we have reached our goals. However, the general case is very difficult to study, because of the peculiar structure of the operators involved in the equations. Indeed, the fragmentation operator acts only on the size of the particles and the space variable is seen as a simple parameter. On the other hand, the diffusion term is essentially a laplacian with respect to the space variable, but includes a diffusivity which depends on the size of the particles in a very singular way. This mixing effects requires new ideas in order to have a complete result.

¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

We have obtained two partial results. The first one deals with a diffusion operator which does not depend on the size variable, the second one allows mixing but we were able to work only at the asymptotic level. We hence plan to continue our study and to generalize our results by adding coagulation effects in the equations.

The scientific mission has been very important because the long time amount spent at the blackboard allowed us to have a deeper scientific exchange and stimulated the emergence of new ideas and strategies. Some of them could be useful also in other contexts.

This STSM contributes to the Action objectives and deliverables specifically in the framework of the WG2, because the research goals are coherent with the goals of the WG2.

We plan to have regular online meetings, at least one per month (November and December 2023, January 2024) and, if necessary, to organize another scientific stay. The results will be described in a scientific article, which we hope to submit before April 2024. The support of the COST action CA18232 will explicitly appear in the paper.