Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Mohamed Fkirine

Details of the STSM

Title: Large time behavior of parabolic Cauchy problems for quantum graphs with vertex noise

Start and end date: 09/09/2023 to 09/10/2023

Description of the work carried out during the STSM

During my Short-Term Scientific Mission (STSM), I focused on the research conducted in the recent paper titled "*On the parabolic Cauchy problem for quantum graphs with vertex noise*" by Mihály Kovács and Eszter Sikolya, published in *Electron. J. Probab. (volume 28, pages 1-20, 2023).* In this paper, the authors delve into the analysis of the heat equation associated with quantum graphs and perturbed vertex conditions. The vertex conditions involve standard continuity and Kirchhoff assumptions at each vertex. By introducing additive Gaussian noise to the Kirchhoff conditions, the authors successfully establish the existence and uniqueness of a mild solution with continuous paths in the standard state space of square integrable functions on the edges. However, the large time behavior of the solutions has not been addressed in the study.

During this STSM, our primary focus was on investigating an important question related to the existence and uniqueness of the invariant measure of this equation.

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Description of the STSM main achievements and planned follow-up activities

Our initial result pertains to the regularity of solutions. We relax the conditions outlined in Theorem 3.8 of the "*On the parabolic Cauchy problem for quantum graphs with vertex noise*" by Mihály Kovács and Eszter Sikolya, published in *Electron. J. Probab. (volume 28, pages 1-20, 2023).* More precisely, we have demonstrated that the unique mild solution of our equation has a version with continuous paths in the Sobolev space H_α, where $\alpha < \frac{1}{4}$, without necessitating the condition (3.14) from Theorem 3.8.

Our second result concerns the investigation of the strong Feller property of the associated transition semigroup. This investigation is directly related to the boundary null controllability of the heat equation on quantum graphs with Kirchhoff boundary control. To tackle this problem, we adopted an approach previously employed by R. D. Russell and H. O. Fattorini in their work *"Exact controllability theorems for linear parabolic equations in one space dimension," published in Arch. Rat. Mech. Anal., 43 (1971), 272-292*, transforming the control problem into an equivalent problem of moments.

Finally, we have provided a comprehensive characterization result for the existence of invariant measures. Surprisingly, we have demonstrated that if an invariant measure exists for our problem, then the transition semigroup cannot be strongly Feller.

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