Report on the outcomes of a Short-Term Scientific Mission[[1]](#footnote-1)

Action number: CA18232 - Mathematical models for interacting dynamics on networks

Grantee name: Annamaria MASSIMINI

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| **Details of the STSM**  Title: Mathematical study of diffusive phenomena in gas particles mixture  Start and end date: 02/05/2023 to 29/06/2023 |
| **Description of the work carried out during the STSM**  Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section. |
| *(max. 500 words)*  Grantee enters max 500 word summary here.  The activity carried out during this STSM can be divided into two parts, pursued in parallel. Firstly, I focused on the general study of kinetic theory, in particular the concept of the hydrodynamic limit, a new research field for me. The second part consisted of weekly meetings with F. Charles and F. Salvarani. In addition to clarifying my doubts, we investigated the type of scaling and model to be adopted to describe the aerosol mixture.  To begin with, we considered an aerosol composed by solid particles (dust), all with the same mass and radius, in a mono-species carrier gas. Both species were assumed to have a spherical shape and to be composed of identical entities.  We described both species by suitable density functions satisfying a system of Boltzmann-like coupled equations, in which four collisional operators encode the species interactions. The precise form of the latter depends on the microscopic behaviour of the collisions. The case of mono and bi-species collisions of the elastic type with hard sphere cross-section is already present in the literature. The advantage of this type of collision is the conservation of mass, momentum, and kinetic energy. We were therefore interested in another scenario: when collisions between gas and dust are of the diffuse reflection type. Since dust particles are macroscopic compared with molecules, the surface of a particle can be considered locally flat from the point of view of an incident molecule. We then assumed that the relative velocity between a particle and a molecule after a collision is given probabilistically by a diffuse reflection on the plane tangent to the particle at the point of impact. In this context, it seems consistent to consider dust particles and molecules as hard spheres coming into contact before the collision. In addition, we had to introduce the surface temperature Tsurf>0 of the dust particles, which complicates a lot the analysis of the model. Indeed, on the one hand, if one assumes that, after colliding with a molecule, Tsurf changes to ensure conservation of total energy (but not kinetic energy), then no equilibria could be found for the distribution functions. If, on the other hand, Tsurf is taken as a constant, there is no conservation of energy, but Gaussian equilibria are recovered. This means that, to close the system, we would perhaps have to add a constitutive law satisfied by Tsurf itself.  Next, to perform a correct asymptotic, we adimensionalize the equations through a proper diffusive scaling. Since the mass ratio between gas molecules and dust particles is very small, we deduced a hierarchy of equations by neglecting some terms, which are multiplied by some small parameters.  As already specified in the application, our project is big and complicated and during this STSM we could only have started. We count on being able to perform the hydrodynamic limit in a rigorous manner, using the method of moments, and then to study the limit system. The next step would be to include different types of dust and repeat the analysis. |
| **Description of the STSM main achievements and planned follow-up activities**  Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.  *(max. 500 words)*  Grantee enters max 500 word summary here.  As far as we are concerned, the STSM has achieved its objectives. Although we have only just started the big project we had in mind, we were able to discuss a lot and I believe that this research stay has fully hit the special focus on Early Career Investigators and gender balance, shared by COST Action. In fact, I was able to lay the foundations for what will be my final PhD thesis project, expand my research network and mathematical knowledge.  Moreover, in the application, we claimed that one of the focuses of this STSM was included in the keywords "coupled systems of evolution equations" and "numerical analysis of coupled PDEs", described in the COST Action MoU. We are satisfied to report that we have succeeded in fulfilling at least one of these two goals. In fact, we have achieved the writing of a coupled Boltzmann-type system, which we expect to become a cross-diffusion system at the limit, when we generalise it to different types of interacting dust. The second goal, namely the numerical analysis of these coupled PDEs, will be a task we will continue in the near future.  Additionally, we have already agreed on plans for future collaborations. In the period from 1 September 2023 to 29 February 2024, I will spend a long research stay in Paris in order to continue to work with F.Charles and F.Salvarani on this project started during the STSM. We hope to be able to rigorously perform the diffusive limit for the system mentioned above, including the multispecies dust case. |

1. This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant. [↑](#footnote-ref-1)