Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Gökhan Mutlu

Details of the STSM

Title: Ubergraphs and Ecological Dynamics

Start and end date: 26/08/2023 to 16/09/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

We have started by reviewing previous approaches that aimed at investigating networks involving higher-order interactions. We were mainly interested in ecological examples in which one species influence or modify the interaction among other species'. Such networks have been usually represented by hypergraphs. We have observed that the previous representations are lacking to reflect the true nature of the underlying networks. We have proposed ubergraphs (a generalisation of graphs in which edges are allowed to be contained in another edges) as a suitable and more general structure to portray higher-order interactions in ecological networks. In an ubergraph, there are fundamental vertices, simple hyperedges (arbitrary subsets of fundamental vertices) which are depth-1 uberedges, depth-k uberedges (k>1) which represents higher-order interactions. We have introduced the set of vertices as a union of set of fundamental vertices and the set of uberedges that are contained in another uberedge. Moreover, we have presented a digraph incidence representation (or a Levy digraph) of an ubergraph as a multilayer digraph such that each layer contains uberedges with same depth. We have shown that there is a one to one correspondence between an ubergraph and its Levy digraph.

Since directions and weights are necessary to fully describe the underlying ecological network, we have introduced directed, weighted ubergraphs. A critical question arises on how to combine weights when we consider the weight of a depth-k uberedge with k>1 (higher-order interactions that are influenced by fundamental vertices or other uberedges). At this stage, we have defined effective weights (the amount of actual flow after the modification) of depth-k uberedges (k>1) recursively as a

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

combination of weights of the modifier and the unmodified interaction. We have shown that this definition is well-defined and represent the flows truly.

Subsequently, we have considered the centrality measures for directed, weighted ubergraphs which have not been introduced in the literature. To this end, we have first defined the incidence and adjacency matrices and then incidence and adjacency degrees of a vertex in a directed, weighted ubergraph. Moreover, we have derived the relation explicitly between incidence/adjacency degree of a vertex in a directed, weighted ubergraph and its Levy digraph. As for path-based measures, we aim to define the notions of shortest path and strongest influence.

Finally, we have compared degree centralities (adjaceny and incidence) of previous hypergraph representation (Golubski et al. 2016) and our ubergraph representation for coffee agroecosystem example which contains higher-order interactions and interaction modifications.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

This STSM enabled me to collaborate in person with two other WG3 members (Aleksandra Puchalska and Mateusz Iskrzynski) as a continuation of NANT mini-project “Analysis of aquatic food webs based on weighted empirical networks”. The mini-project has yielded a nice virtual collaboration since December 2022. Thanks to this STSM, we were able to spend more time and work more efficiently on our research project by discussing in person.

During this STSM, we were able to introduce a suitable framework (directed, weighted ubergraphs) in order to represent complex ecological networks involving higher-order interactions more efficiently than existing representations. This structure also generalizes previous representations such as weighted digraphs and dihypergraphs. We have also achieved to define necessary notions in order to study ecological dynamics such as incidence/adjacency matrices, weights of higher-order interactions which reflect the true amount of flow in the presence of modifiers, incidence/adjacency degrees and paths. All in all, I believe this STSM has been successful in order to reach its objectives.

At the end of the STSM, we have decided to continue our collaboration by meeting online on a regular basis in order to advance our research. We have also agreed to submit our results to a respectful ecology or interdisciplinary journal after defining path-based measures for directed, weighted ubergraphs and comparing these measures with previous hypergraph representations for specific ecological examples involving higher-order interactions such as coffee agroecosystem. Moreover, for future collaboration, we have planned to generalise our results to cover more complex networks involving higher-order interactions rather than just focusing on ecological networks.

Overall, this STSM has contributed to following Action objective and deliverables:

- Coordinating and directing research efforts by groups from different subdisciplines of mathematics,
- Studying evolution equations on higher-dimensional multi-structures or ramified spaces, and developing the abstract theory needed for this study,
- Refining the previously known abstract techniques and developing new ones so that more general DSNs can be modelled and analysed using mathematical tools,
- Establishing an efficient and lasting network of researchers studying DSNs across Europe,
- Fostering the exchange between ITC and non-ITC researchers, with a special focus on Early Career Investigators.