Report on the outcomes of a Short-Term Scientific Mission[[1]](#footnote-1)

Action number: CA18232

Grantee name: Sedef ÖZCAN

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| **Details of the STSM**  Title: Interplay of geometry and observability of magnetic Schrödinger semigroups  Start and end date: 15/10/2023 to 21/10/2023 |
| **Description of the work carried out during the STSM**  Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.  *(max. 500 words)*  Grantee enters max 500 word summary here.  S. Özcan (Dokuz Eylul University, Turkey) visited M. Täufer (University of Hagen, Germany) during the STSM in order to initiate a new joint research project by members of FernUniversität in Hagen, Germany and Dokuz Eylul University in the context of the applicant’s PhD project.  We had formulated four research questions and intended to achieve progress on at least two of them.  As for the first question, we managed to generalize magnetic Bernstein inequalities to three dimensions. While many steps resembled the two dimensional magnetic Bernstein inequality, unexpectedly, proving it in three dimensions required significantly more time and effort. In contrast to the two-dimensional scenario, where the emergence of powers of the Landau operator in Bernstein inequalities was a surprising and natural occurrence, in the case of three dimensions, it necessitated the utilization of appropriate estimates. It necessitated a process of trial and error before we came to the realization that obtaining powers of the Landau operator was not a straightforward approach, and instead, we needed to delve into the understanding of magnetic Sobolev spaces which, in a certain aspect, can be considered an analogous counterpart to the Schwarz space in two dimensions.  We later reviewed the remaining portions of the proof that needed modification. We had extensive discussions, particularly regarding the analytical part. We also meet A. Seelman, who is at Dortmund University of Technology. He possesses deep knowledge about the Logvinenko-Sereda type theorems, to discuss the remaining parts of the proof. We also had a fruitful discussion with A. Seelmann about using a similar technical approach for Schrödinger operators with potentials. This might lead to some progress on the notoriously hard problem of generalizing the Logvinenko-Sereda theorem to nonzero potential.  We also discussed and performed some calculations for higher-dimensional Schrödinger operators with constant magnetic field. The difficulty beyond the two- and three-dimensional case is that more than the Lie algebra of magnetic derivatives will necessarily include more than one nontrivial commutator relation.  We discussed some consequences such as optimal criteria for controllability of magnetic heat equations. The proofs uses the existing Lebeau-Robbiano method for controllability of the heat equations. This is the first and most obvious application of the spectral inequality. Indeed, there is an established method, the so-called Lebeau-Robbiano method to torn spectral inequalities into controllability. We also discussed which dependence of the control cost on the geometry we can expect. In three dimensions with a constant magnetic field space itself might not be homogenous but so far, this is not reflected in our final constant.  Finally, I had a chance to meet Professor D. Mugnolo and studied about the torsional rigidity for Laplacian with delta coupling conditions on quantum graphs, which is an almost completed project. |
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| **Description of the STSM main achievements and planned follow-up activities**  Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.  *(max. 500 words)*  We successfully proved the three-dimensional Magnetic Bernstein inequality. Now our goal is to write this theorem in a well-structured manner, so that we can publish it as soon as possible.  We will continue our work by organizing a meeting with Dr. Sedef Karakılıç, who is at Dokuz Eylul University in İzmir, in the near future to better understand the geometric criteria for the magnetic wave equation. We will most likely combine this article with investigations on geometric criteria for controllability and observability for the wave and Schrödinger equation. For this purpose, we are currently planning a visit of M. Täufer to Dr. Sedef Karakılıç (an expert on magnetic Schrödinger operators) and Sedef Özcan in Izmir where we aim to focus on controllability of the magnetic wave equation. |

1. This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant. [↑](#footnote-ref-1)