

Report on the outcomes of a Short-Term Scientific Mission

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by

ABDELAZIZ RHANDI

During my stay at the University of Agadir, in collaboration with Prof. Said Hadd we discuss and finilaze our project on abstract boundary delay systems and application to flow in network with memory. In particular we continue our common project, in collaboration with A. Batkai, M. Kramar-Fijavz, by studying the positivity of the solutions to the following abstract boundary delay equation

$$\begin{cases} \dot{z}(t) = A_m z(t) + Pz_t, & t \geq 0, \\ Gz(t) = Mz(t) + Lz_t, & t \geq 0, \\ z(0) = x, z(s) = \varphi(s) & s \in [-1, 0]. \end{cases} \quad (1)$$

Here, the operator $A_m : Z \subset X \rightarrow X$ represents a closed operator on the Banach space X , where Z is continuously and densely embedded in X . The linear operators G and $M : Z \rightarrow U$ (with U being a boundary Banach space) are involved, along with the delay operators:

$$Pf = \int_{-1}^0 d\mu(\theta)f(\theta), \quad Lf = \int_{-1}^0 d\nu(\theta)f(\theta), \quad f \in W^{1,p}([-1, 0], X),$$

where $\mu : [-1, 0] \rightarrow \mathcal{L}(X)$ and $\nu : [-1, 0] \rightarrow \mathcal{L}(X, U)$ are functions of bounded variation continuous at zero, with $\mu(0) = \nu(0) = 0$. The initial conditions are given by $x \in X$ and $\varphi \in L^p([-1, 0], X)$.

Using the positivity and the explicit expression of the resolvent of the operator associated to (1) we deduce some important spectral properties and study the asymptotic behaviour of the solution to (1). Applications to a network systems with delays is given.

We expect that an article summarizing the obtained results (which we are currently working on) will be ready for submission.