Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Sahiba Arora

Details of the STSM

Title: Study of the Ornstein-Uhlenbeck semigroups on compact infinite graphs - Part I

Start and end date: 24/04/2022 to 30/04/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

The purpose of this STSM was to start a collaboration with the host Prof. Dr. Abdelaziz Rhandi. During my stay, I also had the opportunity to interact with other members of the Department of Mathematics at the University of Salerno, in particular, Dr. Federica Gregorio and Dr. Christian Tacelli. Moreover, I shared my office with Dr. Jochen Glück, who was a guest at the department for a short while during my stay. Together, we were able to start a new project related to the eventual positivity of operator semigroups.

At the beginning of my stay, we discussed the techniques used by Mugnolo and Rhandi to study the semigroup generated by the Ornstein-Uhlenbeck operator on star graphs and tried to generalize this to the case of trees. However, it became clear quite early that this approach does not work and instead we decided to focus only on rooted trees. In order to study the Laplacian on rooted trees, Naimark and Solomyak used the decomposition of the state space into symmetric and anti-symmetric parts. We studied their techniques and tried to adapt them to our setting.

Furthermore, we discussed the eventual positivity of (the semigroup generated by) a fourth-order operator with unbounded coefficients, whose properties were quite recently studied by Gregorio and Tacelli. Our strategy was to combine their results along with that of the present grantee in order to obtain local eventual positivity of the semigroup.

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<u>Description of the STSM main achievements and planned follow-up activities</u>

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

For (infinite) rooted trees, we recognized that the methods of Naimark and Solomyak are better suited to study the Ornstein-Uhlenbeck operator. Currently, we try to show that the decomposition of the state space into the symmetric and anti-symmetric parts reduces the Ornstein-Uhlenbeck operator, as it was done for the Laplacian in "K. Naimark and M. Solomyak: Geometry of the Sobolev spaces on the regular trees and Hardy's inequalities, Russ. J. Math. Phys. 8, 322–35 (2001)". With this equivalence established, we can then obtain the information about the spectrum using the techniques of the paper "M. Solomyak: On the spectrum of the Laplacian on regular metric trees, Waves in Random Media (2004)" mutatis mutandis. We expect that an article concerning the Ornstein-Uhlenbeck semigroup will be ready for submission in a few months.

For the fourth-order operator with unbounded coefficients, we were able to establish all but one hypothesis of the criterion for individual local eventual positivity given in "S. Arora: Locally eventually positive operator semigroups, To appear in J. Operator Theory". In particular, it remains unclear whether the first eigenfunction is a quasi-interior point. In order to tackle this, we plan to use abstract results (for self-adjoint operators) to show that the spectral bound is a simple pole of the resolvent, which would then allow us to show eventual positivity in nodal domains.