Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Aleksandra Puchalska

Details of the STSM

Title: Generalised impulsive transport equation

Start and end date: 20/09/2023 to 01/10/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

The meeting in Valenciennes was the second (counting also the meeting in Hagen when the collaboration was initialised) meeting when all three collaborators (STSM applicant Aleksandra Puchalska, STSM host Serge Nicaise and another member of a COST Action - Agata Lonc) could discuss together in person the problem described in the proposal.

The first step of the collaboration was discussing the notation and assumptions that had been imposed to assure well-possedness of the generalised impulsive transport model during the on-line discussions. It allowed for better understanding of considered problems and indicated first minor challenges that had to be faced such as the problem of discontinuities' propagation within neighbouring sectors.

We also formulated the procedure that allows for proving the existence of piecewise continuous solution under the assumptions on the topology of neighbouring sectors of propagation. Furthermore, the first conclussions have been drawn to relate restrictions on well-possedness with the dynamical transformations of metric graphs in the corresponding network transport problem. The exact formulation of this relation still needs to be written down.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Let us conclude on the problems that were declared to be conducted during the STSM:

- What are the conditions that allow well-posedness of the generalised impulsive transport equation?
  The conditions in the case of a finite family of impulsive curves are specified.

- How to construct the piecewise-continuous unique solution to the problem?
  The procedure has been developed.

- How to define the class of the networks for which one-to-one correspondence between NT and ITE can be established?
  The first class of dynamical networks is specified.

- What are the perspectives of a new approach to the transport network problem
  The new approach generalises current results and gives an interesting perspective on the relation between discontinuity of the solution and the convexity of impulsive curves.

The collaboration within STSM proved to be fruitful and it will be continued remotely in order to prepare good quality scientific paper. Furthermore, the visit of grantee at the Université Polytechnique Hauts-de-France allows to establish new contacts with local specialists working on PDEs on networks, Felix Ali Mehmeti and Virginie Regnier. It goes in line with the COST Action's objectives since grantee is Female Early Career Investigator from ITC country learning from non-ITC researchers. Furthermore, the topic of collaboration bases on the development of theoretical results joining two mathematical branches, one of them being at the core of the Action interest (transport equation on networks).