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IN SCIENCE & TECHNOLOGY

Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA18232

Grantee name: Marianna Porfido

Details of the STSM

Title: Kernel estimates for some fractional Kolmogorov operators

Start and end date: 16/05/2023 to 30/05/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

I made initial contact with the host, Prof. Said Hadd, and the research group at the Department of Mathematics of the University Ibn Zohr Agadir (Morocco).

During my stay, we investigated kernel estimates for fractional Kolmogorov operators by means of weighted Nash inequalities. I mainly focused my attention on articles such as "*A. Bendikov and P. Maheux, Nash type inequalities for fractional powers of non-negative self-adjoint operators, Trans. Amer. Math. Soc. 359 (2007), 3085-3097*" and "*D. Bakry, F. Bolley, I. Gentil, and P. Maheux, Weighted Nash inequalities, Rev. Mat. Iberoam. 28 (2012), no. 3, 879-906*". In the first paper, starting from a non-negative self-adjoint operator A which satisfies a Nash type inequality, the authors succeeded in obtaining a Nash type inequality for the fractional operator A^{α} ($\alpha > 0$) as well. In the second one, kernel estimates for symmetric Markov semigroups are proved thanks to weighted Nash inequalities. The idea was to generalize the first article to weighted Nash inequalities and apply the results of the second paper to the fractional operator A^{α}.

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

In conclusion, adapting the techniques mentioned above, we were able to prove pointwise bounds for the kernel associated to fractional operators. Moreover, we applied our result to the operator $A = \Delta + \nabla \log \rho \cdot \nabla$, where ρ is a positive smooth function.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

We expect that an article summarizing the obtained results (which we are currently working on) will be ready for submission. This experience will open future collaboration with the research group in Agadir.

Moreover, the work may be applied to fractional operators on networks.

The STSM grantee
Dr. Marianna Porfido
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Date: June 06, 2023

Signature

Marianna Porfido

The host
Prof. Said Hadd
University Ibn Zohr Agadir
Morocco

Date: June 06, 2023

Signature

STSM by Marianna Porfido (University of Salerno, Italy)

Kernel estimates for some fractional Kolmogorov operators

at the Department of Mathematics of the University Ibn Zohr Agadir, Morocco
16 May to 30 May 2023

Background information. Solutions of evolution equations arising from parabolic problems can often be described by strongly continuous semigroups given through a kernel.

In the Euclidean space $\mathbb{R}^n$, the classical Nash inequality was introduced by J. Nash in 1958 to obtain regularity properties and classical uniform bound on the solutions to parabolic partial differential equations. This inequality may be stated in the general framework of symmetric Markov semigroups in L^2-spaces. In addition to the semigroup property and the strong continuity, a symmetric Markov semigroup is a family of positive preserving operators acting on bounded measurable functions, which preserve constant functions and are symmetric on L^2. As proved in [1], weighted Nash inequalities lead to pointwise bounds for the kernel.

In the past years, more and more attention is been given to fractional operators. Given a non-negative self-adjoint operator A, one may define A^α by means of the spectral theorem. In [2], starting from a non-negative self-adjoint operator A which satisfies a Nash type inequality, the authors succeeded in obtaining a Nash type inequality for the fractional operator A^α (for $\alpha > 0$) as well.

Goals of the Short Term Scientific Mission. During my stay at the Department of Mathematics of the University Ibn Zohr Agadir, I worked with the host, Prof. Said Hadd, and with Prof. Abdelaziz Rhandi. The goal was to investigate kernel estimates for fractional Kolmogorov operators by means of weighted Nash inequalities.

Results of the Short Term Scientific Mission. The first step was to improve the results in [2] for weighted Nash inequalities. With this inequality at hand, we applied the technique in [1] in the fractional setting in order to obtain kernel estimates for fractional operators. Finally, we applied our result to the operator A^α, where $A = \Delta + \nabla \log \rho \cdot \nabla$, ρ is a smooth positive function and $0 < \alpha < 1$.

References

[1] D. Bakry, F. Bolley, I. Gentil, and P. Maheux, Weighted Nash inequalities, Rev. Mat. Iberoam. **28** (2012), no. 3, 879-906.

[2] A. Bendikov and P. Maheux, Nash type inequalities for fractional powers of non-negative self-adjoint operators, Trans. Amer. Math. Soc. **359** (2007), 3085-3097.