Report on the outcomes of a Short-Term Scientific Mission[[1]](#footnote-2)

Action number: CA18232 (Mathematical models for interacting dynamics on networks)

Grantee name: Matthias Täufer

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| **Details of the STSM**  Title: Controllability and Non-Controllability on Metric Graphs  Start and end date: 18/02/2023 to 25/02/2023 |
| **Description of the work carried out during the STSM**  Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section. |
| M. Täufer (University of Hagen, Germany) visited I. Nakic (University of Zagreb, Croatia) during the STSM in order to conduct joint research activities.  We had formulated five research questions and intended to achieve progress on at least two of them.  As for the first research question, we hoped to prove that on metric trees, having control on every path connecting two degree-one vertices, one has controllability of the heat equation. We looked at the paper “Control of hyperbolic and parabolic equations on networks and singular limits by Jon Asier Barcena-Petisco, Marcio Cavalcante, Giuseppe Maria Coclite, Nicola de Nitti and Enrique Zuazua” which we intended to generalize. We decided that the methods used therein (Carleman estimates) are not sufficiently flexible and instead resorted to a new approach, combining classical results on controllability on intervals by Russell from the 1980s, abstract control theoretics frameworks (in particular extrapolation spaces) and very recent refinements of the so-called moment method. We worked for three days and arrived at a draft of a simplified proof of controllability on metric trees, which might be interesting in its own right since it yields itself more easily to generalizations such as the presence of loops in the graph and more general boundary conditions.  While we are still in the process of consolidating our proof, we believe that the new method is sufficiently versatile to enable us to tackle research questions 3 (estimates on the control cost), 4 (control on graphs with loops), and 5 (analogous results for the Schrödinger equation) with only minor effort.  We also devoted some time discussing question 2 (“*Does the presence of cycles (or paths between leaves) on which no control is supported always lead to non-controllability?*”). We learned that this is somewhat related to the question whether the heat equation on an interval can be controlled from a single point, which has a surprisingly rich answer that also depends on the “irrationality” of the position of the control point with respect to the endpoints. We now have reason to believe that the answer is positive for the Schrödinger equation, but very much non-trivial for the heat equation and the answer will highly depend on the degree of irrationality present in the graph.  Finally, we also profited from a two-day visit of M. Kramar-Fijavž (University of Ljubljana) with whom we discussed in more detail some aspects of controllability and the effect of surgery procedures in metric graphs on the operators, resolvents and semigroups. This was not only helpful in order for our work on research question 1, but we also initiated a new research project describing changes to the geometry of a metric graph as finite-rank perturbations of the corresponding differential operator which we believe can be developed into an interesting and useful tool in its own right. |
| **Description of the STSM main achievements and planned follow-up activities**  Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.  We have achieved our goals during the STSM. On the one hand, we made progress, developing a new proof of controllability of the heat equation on metric trees, initiated a three-author project (M. Kramar-Fijavž, I. Nakić, M Täufer) and finally, we understood the phenomenon of non-controllability better.  We plan to publish our new proof of controllabililty of metric trees fairly quickly, that is until April 2023, since we are aware of other groups pursuing similar research directions. Therefore, we will also defer follow-up questions which might become accessible due to our method (research questions 3-5) to future papers.  The discussions on non-controllability and the joint project with M. Kramar-Fijavž will likely require future meetings. A Workshop in April 2023 in Hagen, supported by the CA 18232, where I. Nakić and M. Täufer will be present, will provide the next opportunity for further discussions. |

1. This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant. [↑](#footnote-ref-2)