Report on the outcomes of a Short-Term Scientific Mission[[1]](#footnote-1)

Action number: CA 18232

Grantee name: Katarina Bogdanović

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| **Details of the STSM**  Title:Positive kernel operators, operator monotone functions, Laplace transformers and operator semigroups  Start and end date: 17/05/2022 to 07/06/2022 |
| **Description of the work carried out during the STSM**  Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.  Our work consisted of two parts, the study of the spectral theory of kernel operators and strongly continuous semigroups and the research on problems of operator monotone functions, Laplace transforms and Laplace transformers in context of strongly continuous semigroups.  During the work on problems of positive kernel operators we investigated positive kernel operators on Banach function and sequence spaces and problems of positive kernel operators on L^2(X, m) and l^2(R). We studied results from [P] and using results from our previous collaboration [BP] we worked on generalization of some inequalities established in [P] to inequalities for the generalized and the joint spectral radii for bounded sets of positive kernel operators on Banach function and sequence spaces, particularly we were focused to obtain additional results for L^2(X, m) and l^2(R).  The object of our research was [Sh] where is expected to obtain extensions in context of absolute integral operators.  On the other hand the research of the operator monotone functions and problems of Laplace transformers included work on different types of norm inequalities. In order to explore that kind of problems we studied some recently obtained results from [JKL], [JL] and [JL1] and used some recently obtained Cauchy-Schwarz type norm inequalities.  References:  [P] A. Peperko, Inequalities on the spectral radius, operator norm and numerical radius of the Hadamard weighted geometric mean of positive kernel operators, Lin. Mult. Algebra 67:8 (2019), 1637-1652.  [BP] K. Bogdanović, A. Peperko, Hadamard weighted geometric mean inequalities for the spectral and essential spectral radius of positive operators on Banach function and sequence spaces, Positivity (2022) 26:25  [BP1] K. Bogdanović, A. Peperko, Inequalities and Equalities on the joint and generalized spectral and essential spectral radius of the Hadamard geometric mean of bounded sets of positive kernel operators, Preprint  [Sh] S. –Q. Shen, T.-Z. Huang, Several inequalities for the largest singular value and the spectral radius of matrices, Math. Inequal. Appl. 10 (4) (2007), 713-722.  [JL] D. Jocić, M. Lazarević, Cauchy-Schwarz norm inequalities for elementary operators and inner product type transformers generated by families of subnormal operators, Mediterr. J. Math. (2022), 19:49, <https://doi.org/10.1007/s00009-021-01919-x> .  [JL1] D. Jocić, M. Lazarević, Laplace transformers in norm ideals of compact operators, Banach J. Math. Anal. (2021) 15 (4), <https://doi.org/10.1007/s43037-021-00149-3> .  [JKL] D. Jocić, Đ. Krtinić, M. Lazarević, Inequalities for generalized derivations of operator monotone functions in norm ideals of compact operators, Linear Algebra Appl. 586 (2020), 43-63. |
| *(max. 500 words)*  Grantee enters max 500 word summary here. |
| **Description of the STSM main achievements and planned follow-up activities**  Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.  During our STSM planned goals on generalizations theorems for positive kernel operators on Banach function spaces from [P] were obtained. There were found generalizations for bounded sets of positive kernel operators for theorems 3.1 and 3.6, as well as refinements of some considered inequalities. In case of L^2(X, m) and l^2(R) the expected generalizations of theorems 4.6 and 4.8 for bounded sets of positive kernel operators and bounded sets of matrices that define operators were obtained. For new versions of theorems some additional results, that were obtained as well, were needed. This results are expected to be published together with the expected results on extensions some theorems from [Sh] in context of absolute kernel operators. On that area future follow-up collaboration on obtaining extensions of some results from [H] to results for absolute kernel operators were agreed.  For the second part of research, the results concerning a new class of norm inequalities for various types of unitarily invariant norms were obtained, and it is expected to be published soon.  References:  [P] A. Peperko, Inequalities on the spectral radius, operator norm and numerical radius of the Hadamard weighted geometric mean of positive kernel operators, Lin. Mult. Algebra 67:8 (2019), 1637-1652.  [BP] K. Bogdanović, A. Peperko, Hadamard weighted geometric mean inequalities for the spectral and essential spectral radius of positive operators on Banach function and sequence spaces, Positivity (2022) 26:25  [BP1] K. Bogdanović, A. Peperko, Inequalities and Equalities on the joint and generalized spectral and essential spectral radius of the Hadamard geometric mean of bounded sets of positive kernel operators, Preprint  [Sh] S. –Q. Shen, T.-Z. Huang, Several inequalities for the largest singular value and the spectral radius of matrices, Math. Inequal. Appl. 10 (4) (2007), 713-722.  [JL] D. Jocić, M. Lazarević, Cauchy-Schwarz norm inequalities for elementary operators and inner product type transformers generated by families of subnormal operators, Mediterr. J. Math. (2022), 19:49, <https://doi.org/10.1007/s00009-021-01919-x> .  [JL1] D. Jocić, M. Lazarević, Laplace transformers in norm ideals of compact operators, Banach J. Math. Anal. (2021) 15 (4), <https://doi.org/10.1007/s43037-021-00149-3> .  [JKL] D. Jocić, Đ. Krtinić, M. Lazarević, Inequalities for generalized derivations of operator monotone functions in norm ideals of compact operators, Linear Algebra Appl. 586 (2020), 43-63.  [H] M. Hladnik, Absolutely bounded matrices and unconditional convergence, Proc. of the Amercan Math. Society. (2008) 136 (10), 3503-3511.  *(max. 500 words)*  Grantee enters max 500 word summary here. |

1. This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant. [↑](#footnote-ref-1)