

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA18232

Grantee name: Haci Mehmet Guzey

Details of the STSM

Title: Robust Control of Networked Infinite-Dimensional Linear Systems

Start and end date: 29/05/2022 to 11/06/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

The host, Dr. Lassi Paunonen who is working as an Associate Professor in the Department of Mathematics at Tampere University conducts research on the control of PDE systems. My research background is in the control of networked systems. In the early days of STSM, I first explained Dr. Paunonen about my work on the consensus-based control approach. At the same time, I got very useful information from both Dr. Paunonen and his doctoral student about the general mathematical model and control approach of PDE systems. In the following days, we did a literature review on consensus-based networked control of heat equations. Dr Paunonen uses functional analysis-based (semi-group theory) methods in the control of PDE systems. However, we noticed that the networked control of PDE systems, which was developed based on the Lyapunov theory, is also frequently used in the literature. Since my research background is basically based on the Lyapunov theory, we agreed that it is more feasible to proceed in this direction (developing controller through Lyapunov method). In more details, I realized that the Lyapunov function of infinite dimensional systems are defined as L2 norm of the state. The time derivative of the Lyapunov function brings the first and second derivatives of the state with respect to the space. When the time derivative of the state is taken, then integration by parts technique is used. Afterwards, boundary conditions are used. First derivative of the state with respect to the space on one of the boundaries is used as the control input (heat flux).

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¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

As stated in the main description of the COST action; It is quite common that the dynamics arises as a compound effect of the interaction between subsystems in which case we speak about coupled systems. Control of networked PDE systems are relatively new subject which getting more and more attention by the control society. In this context, learning Dr. Paunonen's experience in the control of PDE systems and combining it with my current experience in networked control of ODE systems will yield very productive results.

As a result of the work we have done together during STSM, we have designed controllers in which the 1-D heat equation reaches average consensus, using the Laplacian matrix, and demonstrated the proof of stability through Lyapunov theory. The networked controller also simulated by Dr. Paunonen on MATLAB environment. The simulation results also supported our theoretical results. Our current results are at the level of being published at a conference, and on Dr Paunone's recommendation, we aim to expand the results and publish them in top-tier journals.

In this context, I will work on expanding the controllers we developed with collocated control logic for *non-collocated* cases. For the collocated control, the heat flux is applied to the one side of the boundary and the sensor is also placed to the same point. Since direct temperature sensing at or near the point of control action is often exceedingly hard, temperatures are measured beyond the point of boundary actuation, showing the need for a non-collocated feedback control law. Therefore, non collocated sensing and control has been extensively studied in flexible structures (e.g., [1]), and linear parabolic PDE systems (e.g., [2]).

In addition, control algorithms that will provide average consensus for *nonlinear* PDE systems have also emerged as an alternative direction for us. In addition to these, disturbance rejection and robust connected control of networked PDEs will be investigated in the future.

[1] B.-Z. Guo, J.-M. Wang, and K.-Y. Yang, "Dynamic stabilization of an Euler–Bernoulli beam under boundary control and non-collocated observation," *Systems & Control Letters*, vol. 57, no. 9, pp. 740–749, Sep. 2008. [16]
[2] J.-W. Wang, Y.-Q. Liu, and C.-Y. Sun, "Pointwise exponential stabilization of a linear parabolic PDE system using non-collocated pointwise observation," *Automatica*, vol. 93, pp. 197–210, Jul. 2018.

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