COST Action CA18232 – *Mathematical models for interacting dynamics on networks*

WG2 Meeting – Abstracts

Saint-Cyr-l'École, April 20-22, 2022

Niclas BERNHOFF (Karlstad Univ.)

Compactness results for the linearized Boltzmann collision operators

It is a classical result that the linearized Boltzmann collision operator for single species can be split as the sum of a positive multiplication operator and a compact operator. More recently, those results were extended to multicomponent mixtures. In this talk, we address a new approach to the case of multicomponent gases and discuss extensions to polyatomic gases, as well as some related properties, as Fredholmness and self-adjointness of the linearized collision operators.

Thomas BORSONI (Sorbonne Univ.)

A general framework for the kinetic modeling of polyatomic gases

In the context of kinetic models of polyatomic gases, we propose a Boltzmann framework where the internal structure of molecules is described in a general way, considering the internal states to live in a general measure space. We prove the H theorem in this framework, which gives in particular the equilibrium distribution: the product of a Maxwellian and a Gibbs distribution. We show that this framework contains the monoatomic case, and the two existing polyatomic models, which are the models with continuous internal energy and discrete energy levels. We build new models in this framework directly from physical considerations, which allows in particular to also describe non polytropic gases. Finally, we detail "model reduction", which allows to associate a model with continuous internal energy to any general model. This notably gives a formula to compute the integration weight present in this latter model directly from the description of the molecule. We briefly discuss the extension of this framework to a mixture of gases with chemical reactions.

This is based on joint works with Marzia Bisi and Maria Groppi.

Marjeta KRAMAR FIJAVŽ (Univ. Ljubljana)

The semigroup approach to linear transport on metric graphs

We explain how operator semigroups can be used to study 1D linear Boltzmann equation on a metric graph. We are interested in well-posedness and in the long term behaviour of the solutions to the presented problems.

Julien MATHIAUD (CEA)

Moment methods for rarefied gas dynamics: from theory towards numerics

In this talk, we explore new models for rarefied gas dynamics problems linked to atmospheric reentry. We will construct new models and explain how to solve them.

Jiří MIKYŠKA (Czech Technical Univ. in Prague)

An alternative model of multicomponent diffusion based
on a combination of the Maxwell-Stefan theory and continuum mechanics

We present a theory of multicomponent mixtures which does not employ any splitting of component fluxes into convective and diffusive parts. Instead, momentum balance is formulated individually for each component in which both 1) viscous friction within a component, and 2) momentum exchange among different components, are taken into account. While the viscous friction is described using the Newtonian stress tensor, the Maxwell-Stefan theory is used to describe the momentum exchange among different components. When the viscosity is neglected, the model of ideal mixture of ideal gases leads to a hyperbolic system of conservation laws. For the non-ideal mixtures, we obtain a first-order system in a non-conservative form. A simplified version of the model is discretized using a combination of the finite volume method and the mixed-hybrid finite element method. Numerical examples are provided to show typical behavior of the solution of the model equations.

Marwa SHAHINE (Univ. Bordeaux)
Compactness property of the linearized Boltzmann operator for a mixture of polyatomic gases

In this talk, we present a kinetic description for gas mixtures of polyatomic molecules. We consider a Borgnakke-Larsen-type model of the Boltzmann equation to describe a mixture of polyatomic gases. At the microscopic level, we take into account a continuous microscopic internal energy. Under some convenient assumptions on the collision cross-sections, the linearized Boltzmann operator of this model is Fredholm, by writing it as a perturbation of the collision frequency multiplication operator, which is coercive, and by proving that the perturbation operator is Hilbert-Schmidt.

Quốc Bảo TĂNG (Univ. Graz)
Global existence and equilibration of chemical systems with nonlinear diffusion

In this talk, I discuss the global existence and convergence to equilibrium of chemical reaction systems with nonlinear diffusion. The arbitrarily high order of the reactions as well as the degeneracy of the diffusion make the analysis of such systems challenging. It is shown that if the nonlinear diffusion exponents are large enough with respect to the order of the nonlinearities, global weak or even bounded solutions can be obtained. To study the convergence to chemical equilibrium, we use the entropy method together with a phenomenon called indirect diffusion effect, which indicates that the combination of diffusive chemicals and reversible reactions leads to a diffusion effect on non-diffusive species. Recent developments concerning global renormalized solutions and general equilibration are also mentioned.

This is based on joint works with Klemens Fellner and Evangelos Latos.

Romina TRAVAGLINI (Univ. Parma)
Reaction-diffusion systems derived from kinetic models
for reacting mixtures of monatomic and polyatomic gases

We propose the study of a binary mixture composed by a polyatomic and a monatomic gas, diffusing in a gaseous background (typically, the atmosphere), and undergoing elastic or inelastic scattering, along with reversible and irreversible chemical reactions. We assume that the different interactions between particles occur at different time scales. More specifically, the dominant process is assumed to be the elastic scattering with the host medium, while we present two different scalings for the various chemical reactions. We write the rescaled kinetic Boltzmann equations for the distribution functions of the components and, after suitable integrations, we perform a hydrodynamic limit, obtaining reaction-diffusion equations for the number densities of the constituents. We then discuss the stability properties of these systems, focusing on the occurrence of the Turing instability phenomenon for appropriate choices of particle masses, collision frequencies and, above all, particle internal energies.