Scientific report on my STSM

David Seifert

This report concerns my STSM to visit Professor Serge Nicaise at the Université Polytechnique Hauts-de-France in Valenciennes, France. I travelled from Newcastle to Valenciennes (by train and Eurostar) on Saturday 2 October 2021, and returned to Newcastle (again by train and Eurostar) on Saturday 9 October 2021. The days between Monday 3 October and Friday 8 October 2021 were dedicated to mathematical work on the asymptotic behaviour of coupled systems of differential equations on networks. Professor Nicaise and I were joined for the duration of my visit by my long-term collaborator Dr Lassi Paunonen (Tampere University, Finland).

The objective of this STSM was to initiate a new research project closely related to the topic of our COST Action on *Mathematical models for interacting dynamics on networks*. More specifically, we aimed to study the non-uniform stability properties of strongly continuous semigroups associated with wave-heat systems on graphs, or networks. In such models, one considers a (connected) graph, along each of whose edges one has either a heat equation or a wave equation, and these equations are coupled according to suitable Kirchhoff-type flux and continuity conditions at shared vertices. Such systems may be viewed as simple models of composite fluid-structure interactions, with the wave equations modelling the behaviour of an elastic body (which, on its own, tends to preserve energy) and the heat equations modelling the dissipative effect of a surrounding fluid. Simple wave-heat and wave-heat-wave systems are known to be *polynomially stable* but not exponentially stable, which makes systems of this kind particularly interesting (and challenging) from an operator-theoretic point of view. In this collaboration we tried to gain a more systematic understanding of coupled wave-heat systems on general networks.

The main topic of our discussions in Valenciennes centred on the question of how one might transfer known stability properties of networks of wave equations subject to 'standard damping' at some of the vertices to knowledge about the asymptotic behaviour of the associated wave-heat network, in which damping at vertices is replaced by inserting a heat-edge in an appropriate way. By exploiting the results in a recent paper by L. Paunonen (*SIAM Journal on Control and Optimization*, 2019) and in a recent preprint by L. Paunonen, myself and several other coauthors, we were relatively quickly able to find a first positive result of the desired type. In particular, by combining our new result with previous work by Valein and Zuazua (*SIAM Journal on Control and Optimization*, 2009) we were able to obtain polynomial stability of wave-heat networks with a single exterior heat-edge.

Further discussions led to a certain 'cut-off' and 'lifting' technique inspired by direct PDE methods, which appeared to produce sharper decay results in those cases where it applied. In the final stages of the visit, and in the time that has passed since the end of the visit, we have been writing up our preliminary findings and, simultaneously, trying to modify and improve the abstract framework developed by L. Paunonen in order to obtain optimal rates of decay for general coupled systems. This appears to be on a very good track, and we are moreover optimistic that our general results will be applicable to more general coupled hyperbolic-parabolic systems on networks.

David Seifert

David Seifert
15 October 2021
Newcastle upon Tyne