



COST Action CA18232 -

*Mathematical models for interacting dynamics on networks*

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## WG2 Meeting

Ceptor Andrevlje, Fruška Gora National Park, Serbia

September 20-22, 2021

organized by

Paola Goatin, Bérénice Grec and Milana Pavić-Čolić

local organizers

Milana Pavić-Čolić, Dušanka Perišić and Srboľjub Simić

### Schedule

	Monday September 20	Tuesday September 21	Wednesday September 22
9.15 - 9.30	Welcome / Gathering	Gathering	Gathering
9.30 - 10.15	Goatin	Torrilhon	Madjarevic
10.15 - 11.00	Soares	Briant	Puchalska
11.00 - 11.20	Coffee break		
11.20 - 12.05	Bondesan	Ignat	Mijajlović
12.05 - 12.50	Djordjic	Gregorio	Closure
13.00 - 15.00	Lunch		
15.00 - 18.00	Discussion room at For details how to join please contact the organisers. <i>See the last page for instructions!</i>		

Zoom coordinates:

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For details how to join please contact the organisers.

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## Abstracts

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Andrea Bondesan (Graz University)

*Diffusion asymptotics of the Boltzmann multi-species equation towards the Maxwell-Stefan description*

*Abstract.* We consider the Boltzmann equation modelling monatomic gaseous mixtures. In a standard diffusive scaling where the Mach and Knudsen numbers approach zero at the same rate, we investigate the behaviour of particular solutions expanded around a non-equilibrium Maxwellian state of the mixture whose physical observables solve the Maxwell-Stefan diffusion equations. We make use of the hypocoercive formalism to prove uniform (with respect to the Mach and Knudsen numbers) well-posedness of such solutions, providing a first rigorous derivation of the Maxwell-Stefan model from a kinetic formulation.

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Marc Briant (Université Paris)

*Hypocoercive Techniques in Collisional Kinetic Theory*

*Abstract.* The issue of long-time behaviour of solutions of a PDE, more precisely the convergence towards an equilibrium, can be viewed at a linear(ized) level by adopting a perturbative approach. In such a framework one expects the dynamics of the linear operator to take over higher order terms for small initial data. When the linear operator is symmetric negative then obvious Gronwall-type arguments apply to obtain explicit rate of decay for the solutions. Unfortunately for a lot of collisional kinetic equations, the linear operator indeed offers a negative feedback but only outside of its kernel. In this talk we present different techniques used in order to recover a full coercivity thanks to the interplay between collision and transport operator: operator commutators, weak ellipticity,  $L^2$ - $L^\infty$ , extension methods... We shall present some models where such hypocoercivity proved itself useful to construct explicit Cauchy theories and rates of convergence.

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Vladimir Djordjić (RWTH Aachen University and University of Novi Sad)

*Moment method for a polytropic gas reproducing adjustable transport coefficients*

*Abstract.* In this talk, we will consider the kinetic model of continuous type describing a polyatomic gas in the non-weighted setting. Such a model introduces a single continuous variable supposed to capture all the phenomena related to the more complex structure of a polyatomic molecule.

For the complete polyatomic collision operator we propose a convex combination of purely polyatomic (nonfrozen) and frozen collisions, when a polyatomic molecule behaves as a monatomic one. Motivated by recently proven rigorous existence and uniqueness result in the space homogeneous case, we use the cross section proposed in that analysis and establish macroscopic models. In the case of seventeen moments, we compute relaxation times and transport coefficients in a linearized setting and show their adjustability reflecting the wealth of the model. In particular, we recover both matching with the experimental data for dependence of the shear viscosity upon

temperature and agreement with the theoretical value of Prandtl number given by the Eucken formula.

This work was supported from the COST Action CA18232 MAT-DYN-NET, through the “Short Term Scientific Mission” at RWTH Aachen University.

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Paola Goatin (INRIA Sophia Antipolis - Méditerranée)

*A multi-population traffic flow model on networks accounting for routing strategies*

*Abstract.* We present a macroscopic multi-population traffic flow model on networks accounting for the presence of agents using navigation devices to minimize their instantaneous travel time to destination. The strategic choices of each population differ for the degree of information about the system: while part of the drivers knows only the structure of the network and minimizes the traveled distance, others are informed of the current traffic distribution, and can minimize their travel time avoiding the most congested areas. In particular, the route choices are computed solving Eikonal equations on the road network and they act at road junctions level. The impact on traffic flow efficiency is illustrated by numerical experiments. (Joint work with A. Festa, Polytechnic of Turin.)

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Federica Gregorio (Salerno University)

*Weighted Hardy's inequalities*

*Abstract.* Hardy's inequalities are related to solutions of the heat equation with singular potential  $u_t = \Delta u + \frac{c}{|x|^2} u$ ; the latter being involved in the study of the asymptotic stability of singular steady states of some nonlinear heat equations. We give general conditions to state the weighted Hardy inequality

$$c \int_{\mathbb{R}^N} \frac{\varphi^2}{|x|^2} d\mu \leq \int_{\mathbb{R}^N} |\nabla \varphi|^2 d\mu + C \int_{\mathbb{R}^N} \varphi^2 d\mu, \quad \varphi \in C_c^\infty(\mathbb{R}^N), \quad c \leq c_{0,\mu},$$

with respect to the probability measure  $d\mu$  and optimal constant  $c_{0,\mu}$  which is related to the solution to the following Kolmogorov equation perturbed by a singular potential

$$u_t = \Delta u + \frac{\nabla \mu}{\mu} \cdot \nabla u + \frac{c}{|x|^2} u.$$

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Liviu Ignat (Romanian Academy)

*Asymptotic behavior of solutions for some diffusion problems on metric graphs*

*Abstract.* In this talk we present some recent result about the long time behavior of the solutions for some diffusion processes on a metric graph. We study evolution problems on a metric connected finite graph in which some of the edges have infinity length. We show that the asymptotic behaviour of the solutions of the heat equation (or even some nonlocal diffusion problems) is given by the solution of the heat equation, but on a star shaped graph in which there is only one node and as many infinite edges as in the original graph. In this way we obtain that the compact component that consists in all the vertices and all the edges of finite length can be reduced to a single point when

looking at the asymptotic behaviour of the solutions. We prove that when time is large the solution behaves like a gaussian profile on the infinite edges. When the nonlinear convective part is present we obtain similar results but only on a star shaped tree. This is a joint work with Cristian Cazacu (Bucharest), Ademir Pazoto (Rio de Janeiro), Julio D. Rossi (Buenos Aires) and Angel San Antolin (Alicante).

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Damir Madjarević (University of Novi Sad)

*Multi-Component Mixture of Euler Fluids – Shock Structure Analysis*

*Abstract.* Our aim is to present the solution of the closure problem for a multi-component mixture of Euler fluids with special reference to the shock structure problem. Assuming that the state of the mixture is determined by the fields of mass density, velocity and temperature of each constituent, governing equations consist of the hyperbolic system of balance laws and the closure problem is concerned with determination of the source terms. The problem is solved using combined continuum/kinetic approach: structure of the source terms is determined using continuum approach (compatibility with an entropy inequality), while the phenomenological coefficients are computed by means of kinetic approach (due to compatibility with moment equations for mixtures).

Numerical simulations show that three-component mixtures possess distinguishing features different from the binary ones, and that certain behavior may be attributed to polyatomic structure of the constituents. Shock profiles are strongly dependent on the Mach number, molecular mass ratios of the constituents and their equilibrium concentrations. Mechanical and thermal relaxation times are computed along the shock profiles, and revealed that the thermal ones are smaller in the case discussed in this study.

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Nevena Mijajlović (University of Montenegro)

*Methods for solving quasi-variational inequalities*

*Abstract.* We study methods for solving quasi-variational inequalities which are a notable generalization of the variational inequalities. Solving quasi-variational inequality requires that the corresponding variational inequality be solved concurrently with the calculation of a fixed point of the set-valued mapping. In this paper we suggest and analyze a new continuous and iterative variants of some generalizations of the gradient-type projection method and extra-gradient method.

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Aleksandra Puchalska (University of Warsaw)

*Inviscid Burgers equation on a metric tree*

*Abstract.* Consider the classical inviscid Burgers equation  $u_t + uu_x = 0$  in a monodimensional case. The basic interpretation of the system explains the motion of one wave, creation of shocks and rarefaction waves, but does not capture the interaction of waves, since big waves always eat smaller ones. Hoping to receive passing through phenomenon between waves, think now about the extension of the monodimensional structure of the domain to a graph, giving the possibility for the solution to take different paths.

The above reasoning serves us as a motivation to develop the theory of the Burgers equation on the metric graph. Alike many network problems, the main difficulty here is to determine the behaviour in vertices to make the problem well-posed. In the talk we will show some interesting examples as well as key points of mathematical theory.

The talk is based on joint work with Piotr B. Mucha, University of Warsaw.

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Ana Jacinta Soares (University of Minho)

*On the mathematical modelling of autoimmune diseases*

*Abstract.* Biological systems are usually described at the macroscopic level, by considering the relevant interacting populations that are involved in the system. However, it is well known that a detailed description at the cellular level could be more appropriate in understanding the complex dynamics of many diseases. In particular, the kinetic theory offers a very rich approach, since it is able to describe not only the cellular dynamics and the biological expression of cells but also the global behaviour of the populations by averaging the unknown variables of the kinetic system and passing to the corresponding hydrodynamic limit. In this talk, we present a mathematical model based on a kinetic theory approach for describing the behaviour of autoimmune diseases. We study the mathematical properties of the model and show some numerical results illustrating typical patterns of some autoimmune diseases.

[Work in collaboration with R. Della Marca, M.P. Ramos, C. Ribeiro]

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Manuel Torrilhon (RWTH Aachen University)

*Moment Equations for Rarefied Gas Flow*

*Abstract.* TBA

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## ***Annex. Guide Wonder.me***

All participants are welcome to enter discussion sessions via the wonder.me link provided.

### **Entering the room**

- 1) Open the link on Mozilla Firefox or Google Chrome (this is a browser version and so doesn't require any additional installation!). Please note that Safari does not work.
- 2) Click on 'Request browser permission'.
- 3) Permit Wonder.me to access your camera as well as microphone. Please do not use any further software which requires your camera or microphone such as Zoom Meetings at the same time (kindly close the meeting Zoom Meeting before accessing Wonder).
- 4) Enter the Guest Password
- 5) You can then revise your chosen camera and microphone. If correct, please click 'Next'.
- 6) Take a photo which will be displayed as your avatar. Enter your name and click 'Next'. Your browser will keep your picture and name and you don't need to pass this procedure each time.

### **Moving through the room**

- 1) You can move your avatar through the room with your mouse or arrow keys.
- 2) You can zoom in and out by clicking on '+' and '-' or simply scroll the mouse wheel.
- 3) You can search further participants and invite them directly to a conversation.
- 4) You can chat with further participants.
- 5) You can edit your settings.

### **Interacting with participants**

- 1) Start a conversation by moving your avatar next to another avatar (participant).
- 2) Join a conversation by entering the circle.
- 3) You can invite a participant to join the circle by clicking on its avatar to "invite to circle". The invited person will see the notification and can accept it.
- 4) End a conversation by moving away with your avatar.