Training school on
"Dirichlet Forms on graphs"

Salerno, July 26-28

MAIN COURSES

Introduction to Dirichlet spaces

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We present basic theory of Dirichlet forms and associated geometry. In particular, we discuss regular Dirichlet forms, Beurling-Deny decomposition and intrinsic metrics. We present some key examples and point out some issues of interest.

Dirichlet forms on discrete spaces and analysis on discrete graphs

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We discuss Dirichlet forms on discrete spaces and associated Laplacians on weighted discrete graphs. A particular focus will be laid on the interplay between the geometry of the graph (in terms of intrinsic metrics) and analytic properties such as essential self-adjointness, recurrence, stochastic completeness and spectral properties.

Dirichlet forms and Diffusion equations on metric graphs

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We present metric graphs and diffusion equations on them. In particular, we will focus on describing coupling conditions at the vertices of the graph giving rise to Dirichlet forms. We will also have a look at singular diffusions on metric graphs, and also allow for sticky vertices, which yields models combining discrete and metric graphs.

MINI-TALKS

On the first eigenvalue for the p-Laplacian operators

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This paper is devoted to determine those potential functions which extremize the first eigenvalue under certain conditions for the p-Laplacian operators with Dirichlet boundary conditions. Meanwhile we establish a relation between the first eigenvalue and the real roots of the transcendental equation.

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Overcoming Difficulties concerning Amplitude Equations for Non-self-adjoint Systems

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Global modes and the amplitude equations that describe have been the focus of many authors in Fluid Mechanics. The failure of global modes to describe fluid flows has been related to the non-normality and nonlinearity in the governing equations ([1]). We demonstrate that for strongly non-normal linear operator, it is impossible to capture the saturation characteristics of flow with the global mode governed for the simple reason that, as the flow becomes more non-normal, the projection of the flow onto the global mode becomes less and less of the entire flow. We propose a remedy to this via approximating the solution via a non-self-adjoint perturbation expansion ([2]). The test example that we choose is the non-self-adjoint Ginzburg-Landau equation. We consider this operator with real coefficients and complex coefficients. In the former case, an, albeit unbounded, metric operator generating an underlying lattice of Hilbert Spaces. A Riesz-basis-system therefore exists, which allows us to obtain higher order coefficients correctly with two possible normalisation choices.

References

[1] Chomaz, Jean-Marc. "Global instabilities in spatially developing flows: non-normality and nonlinearity." Annu. Rev. Fluid Mech. **37** (2005): 357-392.

[2] M. I. Vishik, L. A. Lyusternik, "The solution of some perturbation problems for matrices and selfadjoint or non-selfadjoint differential equations. I", Uspekhi Mat. Nauk, 15:3(93) (1960), 3–80; Russian Math. Surveys, 15:3 (1960), 1–73.