WG1 meeting

"C_0-semigroups and beyond"

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ABSTRACTS

Hyperbolic systems on networks – well-posedness, graph-realizability and long term behaviour

Jacek Banasiak
University of Pretoria & Łódź University of Technology
jacek.banasiak@up.ac.za

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In this talk we consider a system of linear hyperbolic differential equations on a network coupled through general transmission conditions of Kirchhoff's type at the nodes. We provide a brief survey of the recent results, discussing the reduction of the problem to the general 1-D hyperbolic system (also called a port-Hamiltonian), a semigroup theoretic proof of the well-posedness of the latter in any L_p space and the availability of explicit solutions leading to the characterization of the long-time behaviour of solutions.

We shall also touch upon the question under which condition a given 1-D hyperbolic system with general Kirchhoff conditions can arise from a problem on a network.
Joint work with A. Błoch.

Some results on second-order elliptic operators with polynomially growing coefficients in L^p-spaces

Sallah-Eddine Boutiah
University Ferhat Abbas Setif-1
boutiah@univ-setif.dz

In this presentation we study the following elliptic operator

$$A_{b,c} = (1+|x|^{\alpha})\Delta + b|x|^{\alpha-1}\frac{x}{|x|}\cdot\nabla - c|x|^{\alpha-2} - |x|^{\beta}$$

defined in the whole $\mathbb{R}^N$. Where $N \geq 3, \alpha \in [0,2)$ and b, c are real numbers.

First, we use quadratic form methods to prove that the minimal realization of $A_{b,c}$ admits an extension that generates an analytic C_0-semigroup for all $p \in (1,\infty)$.

Second, we give conditions on the coefficients under which this extension is precisely the closure of $(A_{b,c}, C_c^{\infty}(\mathbb{R}^N \setminus \{0\}))$.

The case $\alpha > 2$ has been treated in Boutiah et al. (Journal of Differential Equations, (2018) and Discrete Contin. Dyn. Syst. Ser. A. (2019)).

The talk is based on recent work joint with L. Caso (University of Salerno), F. Gregorio (University of Salerno) and C. Tacelli (University of Salerno), Journal of Mathematical Analysis and Applications, 501(2021).

Extrapolation of operator-valued multiplication operators

Christian Budde
North-West University
christian.budde@nwu.ac.za

We study the construction of extrapolation spaces of operator-valued multiplication operators on Bochner L^p-spaces by means of L^p-fiber spaces. We provide some motivation for studying such operator-valued multiplication operators and also first extrapolation spaces of such operators. On the one hand, in operator and spectral theory multiplication operators are a very important class of operators. For example, the spectral theorem for normal operators tells us that for every normal operator on a Hilbert space there exists a multiplication operator representation. Therefore, multiplication operators are closely connected to functional calculus. Another meaningful feature is that multiplication operators occur naturally as Fourier transforms of differential operators with constant coefficients. Even more generally, vector-valued Fourier multipliers are multiplication operators on Banach space-valued function spaces. The stated reasons, a systematic study of multiplication operators and multiplication semigroups on Banach space-valued function spaces as well as their applications are important and already attracted some attention in the past.

On the other hand, extrapolation spaces are useful tools in the context of operator semigroups. They are for example used for the study of maximal regularity or perturbation theory. Special interest has been awakened by the work of Graser [2] who combined the theory of extrapolation spaces with the above mentioned multiplication operators. In fact, he studied extrapolation spaces of operator-valued multiplication operators on the space of all Banach space valued continuous functions vanish at infinity. Graser showed, that the extrapolation of operatorvalued multiplication operators occurs, roughly speaking, fiberwise. We will introduce Banach fiber spaces and fiber integrable functions due to Heymann [3] which were motivated by Banach bundles. We will show that the extrapolation spaces of operator-valued multiplication operators on the spaces $L^p(\mathbb{R}, X)$ of Bochner p-integrable functions are analogous to those studied by Graser. This talk is based on joint work with R. Heymann [1].

References

[1] C. Budde and R. Heymann. Extrapolation of operator-valued multiplication operators. Quaest. Math., 8 pages, 2021. to appear.

[2] T. Graser. Operator multipliers generating strongly continuous semigroups. Semigroup Forum, 55(1):68-79,1997.

[3] R. Heymann. Multiplication operators on Bochner spaces and Banach fibre spaces. PhD thesis, Eberhard-Karls-Universität Tübingen, 2015.

A new hybrid traffic model for traffic flow representation

Roberta Di Pace
University of Salerno
rdipace@unisa.it

Despite the numerous advances in recent years, traffic flow modelling still offers room for improvement in two main respects: the first concerns the level of realism in analysing and describing

traffic flow phenomena, while the second concerns the need to adopt a proper traffic flow model in order to obtain information to be used as input variables for implementing optimisation strategies (prescriptive method). The literature identifies three main approaches to solve the second issue: macroscopic, mesoscopic and microscopic modelling. A further approach has been recently investigated, called hybrid traffic flow modelling, which is also suitable for applications at different scales (multi-scale). The proposed hybrid traffic flow model is based on the combination of two sub-models: an aggregate model (the cell transmission model) and a disaggregate model (the cellular automata model). This hybrid model is developed using a transition cell to connect these sub-models, while the local consistency is discussed.

Perturbed Cauchy problems and applications

Said Hadd
Ibn Zohr University of Agadir
s.hadd@uiz.ac.ma

In this talk, we first discuss generation theorems of some classes of perturbed generators, mainly additive perturbations of the generator (Miyadera-Voigt perturbations) and perturbations of the domain of a generator (Desch-Schappacher and/or Staffans-Weiss perturbations). Second, we give results on regularity of perturbed semigroups such as norm continuity, analyticity, compactness, and positivity. We also discuss the concept of L^p-maximal regularity of perturbed Cauchy problems. We end this talk with perturbed stochastic Cauchy problems.

On a Liouville Theorem for a Ornstein–Ulenbeck Operator

Alessia E. Kogoj
University of Urbino
alessia.kogoj@uniurb.it

We present a one-side Liouville theorem for a Ornstein–Ulenbeck partial differential operator, as a consequence of a Liouville-type theorem at $-\infty$ for the corresponding Kolmogorov operator.

The talk is based on joint work with E. Lanconelli and E. Priola.

On systems of Kolmogorov equations

Luca Lorenzi
University of Parma
luca.lorenzi@unipr.it

In this talk we will survery about some recent results on systems of elliptic and parabolic equations with unbounded coefficients defined in whole $\mathbb{R}^d$. Particular attention will be paid to the L^p-setting.

Boundary null controllability of parabolic systems with dynamic boundary conditions

Lahcen Maniar
Cadi Ayyad University
maniar@uca.ac.ma

In this paper, we study the boundary null controllability of the parabolic equation with dynamic boundary conditions and drift terms

$$\begin{cases} \partial_t y - d\Delta y + c(x)y = f & \text{in } \Omega_T, \\ \partial_t y_\Gamma - \delta\Delta_\Gamma y_\Gamma + d\partial_\nu y + \ell(x)y_\Gamma = g + \mathbb{1}_{\Gamma_0} v & \text{on } \Gamma_T, \\ y_{|\Gamma}(t,x) = y_\Gamma(t,x) & \text{on } \Gamma_T, \\ y(0,\cdot) = y_0 & \text{in } \Omega, \\ y_{|\Gamma}(0,\cdot) = y_{0,\Gamma} & \text{on } \Gamma, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^N$, with smooth boundary $\Gamma = \partial\Omega$ of class C^2, $\nu(x)$ is the outer unit normal field to Ω in the point $M(x)$ of Γ, $\partial_\nu y := (\nu \cdot \nabla y)_{|\Gamma}, d, \delta$ are positive real numbers, $c \in L^\infty(\Omega), \ell \in L^\infty(\Gamma), B \in L^\infty(\Omega)^N, b \in L^\infty(\Gamma)^N, f \in L^2((0,T)\times\Omega)$ and $g \in L^2((0,T)\times\Gamma)$. The function v is a control on a boundary small region $\Gamma_0 \subset \Gamma$.

To achieve our aim, we show first suitable boundary Carleman estimates, which will provide an observability inequality for the backward adjoint problem, which is equivalent to the boundary null controllability of our system.

Bi-Kolmogorov type operators and weighted Rellich's inequalities

Cristian Tacelli
University of Salerno
ctacelli@unisa.it

We consider the symmetric Kolmogorov operator $L = \Delta + \frac{\nabla\mu}{\mu} \cdot \nabla$ on $L^2(\mathbb{R}^N, d\mu)$, where μ is the density of a probability measure on $\mathbb{R}^N$. Under general conditions on μ we prove first weighted Rellich's inequalities with optimal constants and deduce that the operators L and $-L^2$ with domain $H^2(\mathbb{R}^N, d\mu)$ and $H^4(\mathbb{R}^N, d\mu)$ respectively, generate analytic semigroups of contractions on $L^2(\mathbb{R}^N, d\mu)$. We observe that $d\mu$ is the unique invariant measure for the semigroup generated by $-L^2$ and as a consequence we describe the asymptotic behaviour of such semigroup and obtain some local positivity properties. As an application we study the bi-Ornstein-Uhlenbeck operator and its semigroup on $L^2(\mathbb{R}^N, d\mu)$.