

STSM by Michael Kaplin (Ljubljana)

Investigation of relatively uniformly continuous semigroups

at the University of Passau (6 February to 14 February 2020)

part of the COST Action 18232

Background information. Many linear evolution equations that occur in the sciences have the special property that a positive initial value leads to positive solutions for all times. This *positivity property* occurs, for instance, for diffusion and transport processes both on domains and on graphs, for various problems in mathematical biology, as well as in stochastic analysis. The solutions to such problems can often be described by *positive and strongly continuous operator semigroups*.

One drawback of this well-developed theory is its limitation to a single type of time continuity, namely the aforementioned *strong continuity*.

An approach to overcome this limitation was recently developed in the papers [1] and [2], where the authors study *relatively uniformly continuous (ru-continuous) semigroups*; this notion is specifically adapted to settings where positivity occurs.

On many important function spaces, such as L^p for $p \in [1, \infty]$ or spaces of continuous functions, ru-continuity is a stronger property than strong continuity and thus gives us the possibility to study evolution equations whose solutions exhibit improved time regularity. On the other hand, ru-continuous semigroups can also be defined in so-called *vector lattices* that do not carry any topological structure, which opens the theory to describe a wider range of phenomena.

Goals of the Short Term Scientific Mission. During a *Short Term Scientific Mission (STSM)* within the COST Action 18232, Michael Kaplin from the University of Ljubljana (Slovenia), who is one of the initiators of the theory of ru-continuous semigroup, visited Jochen Glück at the University of Passau (Germany) to further develop the theory outlined above.

Two concrete goals of the STSM were (i) to better understand the connection between ru-continuity and strong continuity of semigroups, and (ii) to determine for several important classes of strongly continuous semigroups on function spaces whether they also have the stronger property of being ru-continuous.

Results of the Short Term Scientific Mission. During the STSM, the collaborators found a connection between ru-continuity of semigroups and so called *Gaussian estimates*, which is a classical topic in the study of parabolic partial differential equations. As a consequence, they were able to show that the solution semigroups for large classes of parabolic equations are ru-continuous, even under very weak assumptions in the coefficients of the equation.

Another topic analysed during the STSM are explicit descriptions of ru-continuity on concrete function spaces, in particular on L^p -spaces. On such spaces, there is a close connection between ru-continuity and order continuity, and the latter is, in turn, connected to almost everywhere convergence of functions. The collaborators obtained detailed results about almost everywhere convergence of nets (while this notion of convergence is classically considered for sequences only) and about the description of ru-continuity in terms of almost everywhere continuity.

The results of the STSM will be made available in a research paper which is currently under preparation.

REFERENCES

- [1] M. Kandić and M. Kaplin. Relatively uniformly continuous semigroups on vector lattices. *J. Math. Anal. Appl.*, 489(1):23, 2020. Id/No 124139.

- [2] M. Kaplin and Kramar Fijavž. Generation of relatively uniformly continuous semigroups on vector lattices. *Anal. Math.*, 46(2):293–322, 2020.