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Method of characteristics – improvements

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This work is part of a project about accurate numerical methods for transport processes, which may be used in many applications where transport phenomena are predominant. We aim to design stable and accurate schemes, which are important to provide reliable results. The method of characteristics for solving transport equations is very convenient because it can lead to unconditionally stable numerical schemes. Moreover, it can also naturally ensure the maximum principle. An extension to second order of this method in 1D consists in selecting, at each time and in each cell, a stencil (with two or three points) so that the maximum principle is satisfied. In particular, when two three-point stencils are suitable, the method uses a convex combination of the two schemes corresponding to the two stencils, leading to the introduction of a parameter in each corresponding cell, which can be chosen independently from one cell to the other. The key-point of this work was to assess the influence of the choice for this parameter (in order to reduce numerical diffusion or minimize the loss of mass). This approach was carried out in one spatial dimension with several initial conditions (smooth or not, with compact support or not). We then applied a similar strategy in two spatial dimensions where numerical diffusion and mass loss are more significant. We focus on six-point stencils, which are the most compact ones that can be taken into account to ensure second order. This leads to (up to) three free parameters in each cell that have to be chosen suitably. Since third order cannot be reached with such reduced stencils in 2D, we focus on reducing the diffusive behaviour.