C_0-semigroups and beyond

Conference in honor to Rainer Nagel

25–26 November, 2020

Asymptotic behaviour of positive semigroups

Wolfgang Arendt
University of Ulm, Germany

Abstract. We will give several criteria for the convergence of a positive semigroup (S(t)) as t goes to infinity strongly or in operator norm. Some are classical arguments from Perron-Frobenius Theory developed in the Tübingen school. Our emphasis will be on convergence in operator norm. We present two new criteria (one for semigroups associated with forms and one based on a domination property).

References

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Semi-uniform stability of C_0-semigroups and energy decay of damped waves

Ralph Chill
Tecnical University of Dresden, Germany

Abstract. Semi-uniform stability of C_0-semigroup is a type of asymptotic behaviour which is somewhere in between the more usual exponential stability and strong stability. Due to applications to damped wave equations and many other hyperbolic equations, it has gained considerable attention over the last about 15 years. We give a short and not too technical overview over main results and applications.

ABC for evolution equations on graphs

Klaus J. Engel
University of L'Aquilla, Italy

Abstract. In this talk we apply a recent Weiss-Staffans-type perturbation result to show well-posedness in L^p-spaces of transport-, diffusion- and wave equations on possibly non-compact metric graphs. In contrast to other approaches we are able to treat, simultaneously for arbitrary $1 \leq p < \infty$, general (also non-diagonal) velocity/diffusion matrices and a broad variety of (possibly non-local) boundary conditions. This is joint work with Marjeta Kramar Fijavž.

Time regularity for generalized Mehler semigroups

Alessandra Lunardi
Università di Parma, Italy

Abstract. We study Hölder time regularity in linear evolution equations driven by a large class of differential and pseudo-differential operators L, generators of generalized Mehler semigroups, both in finite and in infinite dimension. Examples of such operators include Ornstein-Uhlenbeck operators with fractional diffusion in finite dimension, and Ornstein-Uhlenbeck operators with associated strong-Feller semigroups, in infinite dimension.

Logaritmic convexity of semigroups and inverse problems of parabolic equations

Lahcen Maniar
University of Marrakech, Morocco

Abstract. In this talk, we present some results on the logaritmic convexity estimate for analytic semigroups. In the case of 2-analytic semigroups, there is an explicit estimate. In the general case, we give a kind of explicit estimate. This estimate is used to study initial data and source inverse problems for an observable abstract system. We illustrate our abstract result by an application to hypoelliptic Ornstein-Uhlenbeck equations.

Functional analytic tools for degenerate PDEs

Giorgio Metafune
Università del Salento, Italy

Abstract. In this talk I will show how functional analytic tools can be employed to prove L^p estimates for parabolic problems associated to degenerate elliptic operators like

$$\Delta + c\frac{x}{|x|^2}\cdot\nabla - b|x|^{-2}$$

with $a > 0$ and b, c real constants. As a consequence, I will discuss L^p estimates for the operator $\Delta_x + D_{yy} + c/yD_y - b/y^2$ in the half-space $y > 0$. This is a joint project with C. Spina and L. Negro.

New characterizations of Kato classes of convolution type Hamiltonians

Mustapha Mokhtar-Kharroubi
University of Besançon , France

Abstract. The role of Kato class potentials appeared in the context of self-adjointness of Schrödinger operators in the classical paper by T. Kato [1]: Their connections with Brownian motion, Harnack's inequality and Lp properties of Schrödinger semigroups were recognized by M. Aizenman and B. Simon [2]. The Kato class was also extended to more general generators of convolution semigroups by R. Carmona, W. Ch. Masters and B. Simon [3]: The object of this talk is to present some results obtained recently [4] on the exploration of this class of potentials in terms of L1 weak compactness properties and also in terms of asymptotics of averages over suitable shells. In particular, various membership criteria are given.

References

1. T. Kato, Schrödinger operators with singular potentials. Israel J. Math, 13, (1972) 135.148.

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Weyl calculus with respect to the Gaussian measure and applications to the Ornstein-Uhlenbeck semigroup

Jan van Neerven
University of Delft, Holland

Abstract. We discuss the Weyl functional calculus $a \mapsto a(Q,P)$ for the "Gaussian position and momentum operators" associated with the Ornstein-Uhlenbeck operator $L = \Delta - x \cdot \nabla$ and show how it can be used to derive some well-known properties of the semigroup generated by L. This talk is based on joint work with Pierre Portal.

Periodicity of Solutions to Evolutions Equations: Massera's and Serrin's Methodologie

Nguyen Thieu Huy
Hanoi University of Science and Technology, Vietnam

Abstract. We present two important methodologies for proving the existence of periodic solutions to evolution equations, which are known as Massera's and Serrin's methods. We combine these methods with one-parameter (strongly continuous) semigroups to obtain interesting results for periodic solutions to parabolic equations (Navier-Stokes equations, Oldroyd - B equations) as well as hyperbolic equations.

The spectrum of the Ornstein-Uhlenbeck operator

Diego Pallara
Università del Salento, Lecce

Abstract. Consider the Ornstein-Uhlenbeck operator

$$A = \mathrm{Tr}[QD^2] + \langle Bx, D\rangle = \sum_{j,k=1}^{n} q_{jk} D_{jk} + \sum_{j,k=1}^{n} b_{jk} x_k D_j, \quad x \in \mathbb{R}^n.$$

Assuming that A is hypoelliptic, i.e., Q_t positive definite for all $t > 0$, where

$$Q_t = \int_0^t e^{sB} Q e^{sB^*}\, ds,$$

I shall describe the spectrum of A in L^p spaces with respect to the Lebesgue measure and the invariant measure

$$d\mu = b(x)dx, \quad b(x) = \frac{1}{(4\pi)^{n/2}(\det\, Q_\infty)^{1/2}} \exp\Big\{-\frac{1}{4} < Q_\infty^{-1} x, x >\Big\}.$$

References

1. G. Metafune, *L^p-spectrum of Ornstein-Uhlenbeck operators*, Ann. Sc. Norm. Sup. Pisa **30** (2001), 97–124.

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