COST Action 18232 - Mathematical models for interacting dynamics on networks

Working Group #4 – Program of the 1st workshop

Zagreb, February 24-26, 2020

Monday, February 24, 2020

10:15	Opening
10:20	Andrea Posilicano
10:45	Coffee break
11:15	Jonathan Rohleder
11:40	Raffaele Carlone
12:05	Joachim Kerner
12:30	Christian Seifert
13:00	Lunch
14:30	Diego Noja
14:55	Open discussion
16:00	Coffee break
16:30	Serge Nicaise

Tuesday, February 25, 2020

Morning	Plenary talks
13:00	Lunch
14:30	Open discussion
16:00	Coffee break
17:00	David Krejčiřík

Wednesday, February 26, 2020

Morning	Open discussion
13:00	Lunch

Abstracts

On the nonlinear Dirac equation on metric graphs
Raffaele Carlone

In this talk I will review some recent results on the nonlinear Dirac (NLD) equation on metric graphs. In the case of localized Kerr nonlinearities, with Kirchhoff-type conditions at the vertices, we discuss existence and multiplicity of the bound states. We also prove that, in the subcritical case, they converge to the bound states of the nonlinear Schrödinger equation in the nonrelativistic limit. In the second part we consider a Kerr-type nonlinearity on non-compact metric graphs with a finite number of edges, in the case of Kirchhoff-type vertex conditions. We prove local well-posedness for the associated Cauchy problem in the operator domain and, for infinite N-star graphs, the existence of standing waves.
This is a joint work with W. Borrelli and L. Tentarelli.

On Pleijel's theorem, an improvement and a conjecture
Joachim Kerner

In this talk I will discuss a well-known theorem of Pleijel from 1956 regarding nodal domains of Laplacian eigenfunctions and its recent improvement by Bourgain in 2013. In particular, we formulate a conjecture of Polterovich from 2009 that one would like to prove or disprove.

Spectral geometry of quantum waveguides
David Krejčiřík

We shall make an overview of the interplay between the geometry of tubular neighbourhoods of Riemannian manifolds and the spectrum of the associated Dirichlet Laplacian. An emphasis will be put on the existence of curvature-induced eigenvalues in bent tubes and Hardy-type inequalities in twisted tubes of non-circular cross-section. Consequences of the results for physical systems modelled by the Schrö dinger or heat equations will be discussed.

Variational methods for quantum graphs and applications to eigenvalue estimates and evolution equations
(as plenary talk)
Pavel Kurasov and **Delio Mugnolo**

Gradient flows in metric random walk spaces
(as plenary talk)
José M. Mazón

In our digital world many different kinds of data are now available (images, social networks, etc). To represent these data the most natural and flexible representation consist in using weighted graphs. Historically the main tools for the study of graphs came from combinatorial graph theory. Recently, there has been increasing interest in the research of partial differential equations (PDE's) in graphs. On the other hand, in recent years there has been a great development of the study of nonlocal PDE's motivated by problems in image processing, the analysis of the peridynamic formulation of the continuous mechanic, Markov jump processes, etc. In the last years we have studied some gradient flows in the general framework of metric random walk spaces, that is, metric spaces together with a probability measure assigned to each point of the space that encode the jumps of a Markov

process, that include, in particular, PDE's in weighted graphs and some nonlocal PDE's. In this lecture we summarize these results

Linear Hyperbolic Systems on Networks
Serge Nicaise

We consider hyperbolic systems of linear partial differential equations on 1-dimensional structures that we interpret as networks. On each edge of the network we take an evolution equation of the form

$$\frac{\partial u_e}{\partial t}(t,x) = M_e(x)\frac{\partial u_e}{\partial x}(t,x) + N_e(x)u_e(t,x), \quad t \geq 0,\ x \in (0,\ell_e), \tag{1}$$

where u_e is a vector-valued function of size $k_e \in \{1,2,3,\ldots\}$, M_e and N_e are matrix-valued functions of size $k_e \times k_e$ and ℓ_e is the length of the edge e. We couple equations (1) for different edges e via boundary conditions. Our goal is to characterise boundary conditions that yield a well-posed problem on the network. We also study some qualitative properties of the solutions.
Our framework covers a large variety of models, like 1D-Maxwell equations in cable networks or linearised Saint-Venant model for water dynamics in sediment-filled canals, for example.
This is a joint work with Marjeta Kramar Fijavž (Ljubljana, Slovenia) and Delio Mugnolo (Hagen, Germany).

Some spectral problems arising in the analysis of nonlinear Schrödinger equation on metric graphs
Diego Noja

The semi-classical limit on a star-graph
Andrea Posilicano

We consider the semi-classical limit of the quantum evolution of quasi-Gaussian coherent states on a quantum star graph with Kirchhoff boundary conditions at the vertex. We show that the quantum evolution can be approximated by the one generated by a suitable selfadjoint extension of the symmetric operator corresponding to the free classical evolution away from the vertex.

A Calderón type inverse problem for quantum trees
Jonathan Rohleder

We solve the inverse problem of recovering a metric tree from the knowledge of the corresponding Dirichlet-to-Neumann matrix on the boundary. This is done by tracing the problem back to an inverse problem for the Schur complement of the discrete Laplacian on an associated weighted tree. Moreover, we provide examples which show that several possible generalizations of this result, e.g. to graphs with cycles, fail. The result complements related inverse problems where the given data is the Titchmarsh-Weyl function, i.e. a parameter-dependent Dirichlet-to-Neumann matrix. This is joint work with Hannes Gernandt.

Monotonicity of Spectra of Schrödinger Operators on Metric Graphs
Christian Seifert

In recent years the dependence of spectra of Schrödinger operators on the geometric or metric parameters of the underlying metric graph has been investigated. We will

present some monotonicity results for various kinds of coupling conditions. This is joint work with Jonathan Rohleder (Stockholm).